TOWARDS A THEORY OF GROUND-THEORETIC CONTENT

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ABSTRACT. A lot of research has recently been done on the topic of ground, and in particular on the logic of ground. According to a broad consensus in that debate, ground is hyperintensional in the sense that even logically equivalent truths may differ with respect to what grounds them, and what they ground. This renders pressing the question of what we may take to be the ground-theoretic content of a true statement, i.e. that aspect of the statement's overall content to which ground is sensitive. I propose a novel answer to this question, namely that ground tracks *how*, rather than just *by what*, a statement is made true. I develop that answer in the form of a formal theory of ground-theoretic content, show how the resulting framework may be used to articulate theories of ground, including in particular an attractive account of the grounds of truthfunctionally complex truths that has proved difficult to accommodate on alternative views of content.

1. INTRODUCTION

Recently a lot of research has been devoted to *ground* – the relation, as Kit Fine has put it, 'of one truth holding *in virtue of* others' (Fine, 2012c: p. 1) – and in particular to the broadly *logical* features of ground.[1] A distinctive feature of ground, according to current consensus, is that it is *hyperintensional* in the sense that even logically equivalent truths may differ with respect to what grounds them and what they ground (cf. (Correia and Schnieder, 2012: p. 14)). For instance, the truth that snow is white is taken to ground the truth that snow is white or snow is not white, but not the logically equivalent truth that grass is green or grass is not green. And the truth that snow is white or snow is not white in turn grounds the truth obtained by adding as a further disjunct

[1] The pioneering contributions initiating this debate were Batchelor (2010); Correia (2010, 2014a); Fine (2010, 2012c,b); Rosen (2010); Schnieder (2011). More recent work includes Correia (2014b); deRosset (2013, 2014); Krämer (2013); Krämer and Roski (2015); Krämer and Roski (2016); Litland (2013, 2016a); Poggiolesi (2015).

the proposition that 2+2=5, which is not grounded by the logically equivalent truth that grass is green or grass is not green.

Ground is accordingly sensitive to features of a truth that go beyond its logical profile, and in particular beyond the matter of what possible worlds the truth obtains in. One pressing question in the development of the theory of ground is therefore just what the features of a truth are that are tracked by ground. For once we have an answer to this question, we may form a notion of a *ground-theoretic content* by abstracting from those features of a truth to which ground is blind. We can then go on to construct a mathematical representation of ground-theoretic content, which may serve as a common framework within which to formally articulate, study, and compare the various competing views of ground.

In this paper I propose and defend a particular answer to the question what ground is sensitive to. My proposal builds on, and modifies, the view implicit in the semantics of ground that Fine has developed in his influential papers 'The Pure Logic of Ground' and 'Guide to Ground' (Fine, 2012c,b). Central to this view is the notion of a *fact* – roughly, a proper or improper part of the actual world – *verifying* a truth. The view then presents the relationship of ground as sensitive purely to mereological relationships between the facts that verify the relevant truths. Unfortunately, as Fine concedes, the view is limited in its capability to serve as a framework for the articulation of plausible theories of ground. For it cannot accommodate certain widely held principles about the interaction of ground with the truth-functional operations of conjunction, disjunction, and negation.

I argue that we can overcome this problem by means of a very natural modification of Fine's approach. The key element of my proposal is the notion of a *mode of verification*, which corresponds to a certain kind of answer to the question *how* a truth is verified by a fact. A disjunction $P \vee Q$, for example, may plausibly be verified either by verifying its first disjunct P, or by verifying its second disjunct Q, and if P and Q are distinct propositions, then these modes of verifying $P \vee Q$ will be distinct as well. I suggest that it is these features of a truth, the modes in which it is verified, that ground tracks. In particular, I take some truths $P_1, P_2, \ldots$ to ground a truth Q just in case Q is verified by verifying $P_1, P_2, \ldots$[2]

The plan for the paper is as follows. Section 2 clarifies and motivates the project of the paper. Section 3 briefly describes Fine's proposal and its limitations. In section 4 I informally introduce the notion of a mode of verification. I go on to show how a plausible account of how conjunctions, disjunctions, and negations are verified combines with

2 A word on notation. I shall be somewhat sloppy in my use of the letters 'P', 'Q', etc., in that I sometimes use them as schematic sentence letters, and sometimes as variables ranging over contents that may be assigned to sentences.

the proposed view of ground to validate the principles that Fine was unable to accommodate. This constitutes a partial vindication of my proposal. A long penultimate section 5 describes a mathematical representation of ground-theoretic content to serve as a formal framework for developing the theory of ground. I then use the framework to define a relation of ground and operations of conjunction, disjunction, and negation. The emerging view is again shown to validate the principles linking ground and the truth-functions that are invalid on Fine's account. Finally, I show how various competing views of the structural principles of ground, as well as of ground-theoretic equivalence, may be implemented within my formal framework. Section 6 concludes.

2. Preliminaries

Following Fine, I have informally spoken of ground as a relation between truths. It is controversial, however, whether this is the best, most perspicuous, or most fundamental way of speaking about ground. There are two worries about this, one targeting the term 'relation', one concerning the term 'truths'. I discuss them in turn.

A number of authors prefer to use a *sentential operator* to express ground, rather than a relational predicate such as 'ground(s)'.[3] Using $<$ as the symbol for ground, they would therefore write 'the ball is round $<$ the ball is red or round' rather than 'the truth that the ball is round grounds the truth that the ball is red or round'. Speaking in this way, it may be argued, no more commits one to a relation of ground holding between some truths than the use of 'if and only if' commits one to a relation of equivalence holding between some propositions.

However, even on this view, it is highly desirable to have a theory of ground-theoretic content of the kind I am after. Let it be granted that there is a legitimate notion of the overall *content* of a sentence. Then we may ask with respect to this notion which of the features of a sentence's content the grounding operator $<$ is sensitive to in the sense that sameness with respect to these features of content guarantees that two sentences may replace one another within the scope of $<$ without changing the truth value.[4] Abstracting

[3] The operator option is chosen, for example, by Fine (2012b), Correia (2010), and Schnieder (2011). The predicate option is preferred by Rosen (2010), as well as Schaffer (2009). (The latter is something of an outlier in the current debate, though, in that he takes ground to relate not just truths, but objects of *any* kind. His conception of ground will not be canvassed in this paper.)

[4] I assume here that ground is sensitive only to differences between sentences that concern content. This is not obvious prior to investigation; it may be that the best account of the distinctions drawn by ground sees (some of) them as purely *syntactic*. For the purposes of this paper, the assumption has the status of a working hypothesis. That is, I propose that we try and see if we can make sufficiently fine-grained distinctions pertaining to content to capture the distinctions drawn by ground. (One potentially problematic kind of case arises in connection with conceptual analyses. It might be suggested that if

again from any other features of a sentence's content, we get our notion of ground-theoretic content. The ground-theoretic content of a sentence will then be exactly what is suited to play the role of a sentence's *semantic value* in formulating a semantics for ground. A formal framework within which to theorize about ground-theoretic content thus provides a framework for the development of semantic theories of ground.

We turn to the second worry concerning the talk of ground as a relation between truths, targeting the use of the term 'truth'. A truth, presumably, is some kind of (accurate) representation of the world, and so ground, if it is a relation between truths, would appear to be a meta-representational relation. A number of authors however prefer to see ground as a relation between *worldly* items rather than representational ones, for which the term 'fact' would then appear more appropriate than 'truth' (cf. Correia (2010: p. 258f)). An alternative option, preferred by Correia in later work, is to simply distinguish two legitimate notions of ground, one worldly, one representational (cf. his 2014a: sec. 5 and 2014b: p. 36).[5] I myself do not find the worldly/representational distinction very helpful or clear. It should be noted, though, that nothing in the way I have introduced the notion of ground-theoretic content hinges on the use of the term 'truth' rather than 'fact', or on taking the relata of ground to be representational entities.[6] So even if the only legitimate notion of ground is somehow worldly, this by itself threatens neither interest nor feasibility of my project.

A clearer distinction than that between worldly and representational items obtains between two alternative kinds of views on the *logical* principles for ground which Correia takes to be characteristic of worldly and representational conceptions of ground. On one

Alice is a vixen, say, then this is so *because* Alice is a female fox, where the 'because' indicates grounding, and that nevertheless 'Alice is a vixen' and 'Alice is a female fox' are exactly alike in content. For discussion, see (Schnieder, 2010). Thanks to an anonymous referee for highlighting the relevance of these cases.)

5 Note that the distinction, if it can be made, may also be transposed to the sentential-operator setting, where it turns into a distinction between worldly and representational conceptions of ground-theoretic content; cf. (Correia, 2010: 257).

6 It may be objected that I treat ground, in effect, as a relation between ground-theoretic contents, and since contents are representational entities, this commits me to a representational conception of ground. If this were so, the same reasoning would reveal Correia's ostensively worldly conception of ground as representational, for he, too, treats ground in effect as a relation between what he calls the worldly contents of sentences. In his intended sense, then, a content may be worldly and thus non-representational. This is one of the reasons why I do not find this way of making the distinction very helpful.

kind of view, the following *introduction* principles for ground are taken to hold without restriction:[7]

($<$I$\vee$): If P then $P < P \vee Q$ and $P < Q \vee P$
($<$I$\wedge$): If P, Q then $P, Q < P \wedge Q$
($<$I$\neg\wedge$): If $\neg P$ then $\neg P < \neg(P \wedge Q)$ and $\neg P < \neg(Q \wedge P)$
($<$I$\neg\vee$): If $\neg P, \neg Q$ then $\neg P, \neg Q < \neg(P \vee Q)$
($<$I$\neg\neg$): If P then $P < \neg\neg P$

Now on all of the extant logics of ground, *typical* instances of the first four of these principles are taken to hold. But opinions diverge with respect to some special kinds of instances. Consider in particular the case in which $P = Q$. If the principles hold even for this case, then for any truth or fact P, we have that P grounds $P \vee P$ as well as that P grounds $P \wedge P$. Given the standard assumption that ground is not generally reflexive, it follows that a distinction must be made between P on the one hand, and $P \vee P$ as well as $P \wedge P$ on the other hand. But such a distinction, Correia claims, is only plausible given a conception of the relata of ground as representational entities, and he takes the same view with respect to the distinction required by the last principle between a truth P and its double negation $\neg\neg P$. On a worldly conception of ground, he concludes, ($<$I$\neg\neg$) is to be rejected, and the other principles have to be restricted to rule out, at least, the case of $P = Q$.[8]

So let us set aside the worldly/representational distinction and instead focus directly on the introduction principles. Should we take them to hold in unrestricted form? Or should they be restricted in some way? Or does it simply depend on which of two equally legitimate notions of ground is at issue? Rather than try to decide the issue in advance of developing a theory of ground-theoretic content, I suggest that well-founded answers should be based in part on how the possible views can be implemented within such a theory. And so the question arises whether each view can be accommodated within an attractive theory of ground-theoretic content.

A partial answer to this question is contained in Fine's work, which provides us with an elegant theory of ground-theoretic content that naturally accommodates the view that the introduction principles should be restricted. For on that account and given Fine's definition of ground, certain relatively natural restrictions of the introduction principles

7 This kind of view is endorsed, for example, by Fine (2012b), and in related forms by Rosen (2010), Schnieder (2011), and Correia (2014b).

8 Cf. (Correia, 2010: p. 267f) – It is plausible that a number of other cases should then also be disallowed; cf. ibid, p. 269.

turn out to hold. The question is then whether it is also possible to formulate an alternative account that equally naturally accommodates the view that the introduction principles should not be restricted; as Fine himself points out, his own account cannot serve in this role. My aim in what follows is therefore to develop such a theory of ground-theoretic content and thereby to establish a positive answer to this second question.

3. The Truthmaker Conception of Content

Fine (2012b) formulates a semantics for ground, and implicit in that semantics is, firstly, an answer to the question to what features of a sentence's content ground is sensitive, and secondly, a theory of ground-theoretic content. This section briefly summarizes Fine's account and identifies its limitations hinted at above.

The semantics Fine proposes associates with each sentence A two sets of facts, namely the set $[A]^+$ of facts verifying the sentence and the set of facts $[A]^-$ falsifying the sentence. Facts are described by Fine in comparison with possible worlds: they are like possible worlds except in that they are all of them *actual*, and in that they are generally *incomplete* in the sense that they leave open the truth-value of some propositions. Moreover, it is assumed that facts may be *fused*, so that given any facts $s, t, u, \ldots$, we may form their fusion $\bigsqcup\{s, t, u, \ldots\} = s \sqcup t \sqcup u \sqcup \ldots$, which contains all of s, t, u as parts. For a sentence to be true is for it to be verified by some fact, and for it to be false is for it to be falsified by some fact. So if we assume that no sentence is both true and false and no sentence is neither true nor false, then for every sentence A, exactly one of $[A]^+$ and $[A]^-$ is empty.

A content P, on this view, may accordingly be identified with a pair $\langle P^+, P^- \rangle$ of two sets of facts, where any member of P^+ is a verifier of P, and any member of P^- is a falsifier of P. Fine assumes that any fusion of verifiers of a sentence also verifies the sentence and similarly for falsifiers. So both components of a content are required to be closed under fusion.

Fine then formulates semantic clauses for four distinct grounding operators; the corresponding relations on contents may be defined as follows:[9]

$P_1, P_2, \ldots \leq Q$	iff $s_1 \sqcup s_2 \sqcup \ldots \in Q^+$ whenever $s_1 \in P_1^+$, $s_2 \in P_2^+, \ldots$
$P \preceq Q$	iff $P, R_1, R_2, \ldots \leq Q$ for some $R_1, R_2, \ldots$
$P_1, P_2, \ldots < Q$	iff $P_1, P_2, \ldots \leq Q$ and $Q \npreceq P_1$, $Q \npreceq P_2$, $\ldots$
$P \prec Q$	iff $P \preceq Q$ and $Q \npreceq P$

[9] Although Fine does not say so, the definitions should be read as restricted to truths, otherwise $P_1, P_2, \ldots \leq Q$ will vacuously hold whenever one of $P_1, P_2, \ldots$ is false.

Somewhat curiously, the most basic notion of ground of the four is not the familiar notion of ground but a *reflexive* notion that Fine calls *weak full ground* ($\leq$). In terms of this notion and its natural *partial* counterpart ($\preceq$), Fine then defines an irreflexive notion of *strict full ground* ($<$), as well as a notion of *strict partial ground* ($\prec$).

Weak full ground, and thereby ultimately all notions of ground, is defined essentially in terms of the notion of fusion and verification. Ground, on this picture, is thus taken to track mereological relationships between the facts verifying the relevant truths, and ground-theoretic content is accordingly taken to be a matter of which facts verify a given truth.

Given this view of ground-theoretic content, however, it does not appear possible to accommodate the unrestricted introduction principles. To see this, we need to ask how the operations of conjunction, disjunction, and negation may be defined on Fine's contents. The view adopted in Fine (2012b) corresponds to these definitions:

$$
\begin{aligned}
(P \wedge Q)^+ &= \{s \sqcup t : s \in P^+ \text{ and } t \in Q^+\} \\
(P \wedge Q)^- &= P^- \cup Q^- \cup \{s \sqcup t : s \in P^- \text{ and } t \in Q^-\} \\
(P \vee Q)^+ &= P^+ \cup Q^+ \cup \{s \sqcup t : s \in P^+ \text{ and } t \in Q^+\} \\
(P \vee Q)^- &= \{s \sqcup t : s \in P^- \text{ and } t \in Q^-\} \\
(\neg P)^+ &= P^- \\
(\neg P)^- &= P^+
\end{aligned}
$$

But this implies that for all P, $P = P \wedge P = P \vee P = \neg\neg P$, and accordingly we never have $P < P \wedge P$, $P < P \vee P$, or $P < \neg\neg P$. Moreover, even independently of these specific clauses, it is hard to see how the problem could be solved within the present framework. For there appears to be no plausible way to distinguish between a truth P on the one hand and the corresponding truths $P \vee P$, $P \wedge P$, or $\neg\neg P$ on the other hand, appealing only to the facts verifying them, and in such a way as to generally render the relevant grounding claims true. The most obvious way to allow for some sort of distinction between P and $P \vee P$ as well as $P \wedge P$ would be to lift the requirement that P be closed under fusion. But this would not help for the case of $\neg\neg P$, and even for $P \wedge P$ and $P \vee P$, it would of course help only in the special case where P is not closed under fusion. If we wish to accommodate the envisaged introduction principles in unrestricted form, we need to modify the Finean framework.[10]

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10 Note that I am relying here on the assumption, noted in footnote 4 above, that ground is sensitive only to differences in content. If this assumption were given up, and ground seen as sensitive to the linguistic guises of contents, then the above line of reasoning could be resisted. Thanks here to an anonymous referee.

4. Introducing Modes of Verification

To see how a suitable conception of ground-theoretic content might be obtained, we adopt for the moment an informal understanding of propositions and ask what distinctions we can then draw between a proposition P and the corresponding disjunction $P \vee P$ that might be tracked by the relation of ground.

Consider first the easier case of the disjunction $P \vee Q$ of the distinct propositions P that Alice is smart and Q that Bob is tall. Suppose the fact s of Alice having an IQ of 150 verifies P and the fact t that Bob is 6 foot tall verifies Q. Then both s and t separately verify $P \vee Q$. But it seems natural to think that there is a *difference* between the verification of $P \vee Q$ by s and the verification of $P \vee Q$ by t, which we can bring out by asking *how* the facts verify the proposition. The fact s verifies the proposition $P \vee Q$ by verifying the disjunct P that Alice is smart. The fact t, in contrast, verifies $P \vee Q$ by verifying its other disjunct Q that Bob is tall.

Thus attending to the ways, or *modes* in which a proposition is verified by its verifiers, also enables us to distinguish between P and $P \vee P$. For it is very natural to say that s verifies $P \vee P$ just as it verifies $P \vee Q$, by verifying P. At the same time, it is not at all tempting to say that s verifies P itself by verifying P. I would suggest, moreover, that it is this distinction which is tracked by the grounding relation. Roughly speaking, P grounds $P \vee P$ precisely because $P \vee P$ may be verified by verifying P, whereas P does not ground P, since P may not be verified by verifying P.

Before we move on to consider the case of conjunction, a brief comment on my use of 'by' is required to avoid misunderstanding. As used in ordinary language, a 'by'-statement can be true even if it provides only a *partial* answer to the corresponding 'how'-question. For instance, we may say that someone got into the house by breaking a window without thereby implying that breaking the window was on its own sufficient for the person to get into the house. Even if it was also required that the person climb through the window, the 'by'-statement may express a truth by ordinary language standards. I wish to highlight, therefore, that the uses of 'by' I make in the formulation of my proposal, in contrast, *are* to be understood as imposing the kind of *fullness* or *sufficiency* condition that is lacking in the ordinary reading of 'by'.[11]

We turn now to the case of P and $P \wedge P$. Again, we first consider the easier case of the conjunction $P \wedge Q$ of the distinct propositions P and Q. Since s verifies P and t verifies Q, their fusion $s \sqcup t$ verifies $P \wedge Q$. But *how* does $s \sqcup t$ verify $P \wedge Q$? A natural first thought is: by verifying P and Q. However, on the perhaps most natural, *distributive* reading

[11] Cf. (Schnieder, 2008: p. 665f.). As Schnieder points out elsewhere (2011: 450f), similar cautionary remarks are in order when locutions such as 'because' are used to convey relationships of full ground.

of that suggestion, it implies that $s \sqcup t$ verifies *each* of these two propositions. And this is typically not the case, for on the Finean view, a verifier of a conjunction does not in general verify each, or even just one of the conjuncts.[12] The difficulty can be resolved by employing a *non-distributive* interpretation of the claim that the propositions P and Q are verified by a fact like $s \sqcup t$.[13] The idea is that in the relevant sense, the propositions $P_1, P_2, \ldots$ are verified by a fact iff the fact is the fusion of some facts $s_1, s_2, \ldots$ verifying $P_1, P_2, \ldots$ respectively. Since s, t verify P and Q, respectively, their fusion $s \sqcup t$ verifies, in the non-distributive sense, P and Q. Moreover, I claim that it is *by* verifying P and Q that $s \sqcup t$ verifies $P \wedge Q$.[14]

So by taking into account the modes in which a proposition is verified, we are also able to distinguish between P and $P \wedge P$. For while s verifies $P \wedge P$ by verifying P, it is not the case that s verifies P by verifying P. And as before, P may be taken to ground $P \wedge P$ on the strength of the fact $P \wedge P$ may be verified by verifying P, whereas P does not ground P, since P may not be verified by verifying P. More generally, the account of ground I propose may in a first approximation be stated thus:

(<): $Q_1, Q_2, \ldots < P$ iff
(i) $P, Q_1, Q_2, \ldots$ are true, and
(ii) every verifier of $Q_1, Q_2, \ldots$ verifies P by verifying $Q_1, Q_2, \ldots$

Consider now finally the negative proposition $\neg Q$ that it is not the case that Bob is tall. How should we take it to be verified? Like Fine, I adopt a *bilateral* conception of content to account for negation. On this conception, a content encodes information concerning both how it is verified and how it is falsified. So in giving an account of $\neg Q$, we may appeal to both what verifies its negatum Q, and what falsifies it. Again with Fine, I take it that $\neg Q$ is verified by exactly those facts that falsify Q, and that $\neg Q$

[12] This illustrates that Fine's notion of verification is non-monotonic in the sense that a fact may fail to verify a proposition even though it has a part which verifies the proposition. For this reason, Fine sometimes describes this notion of verification, which he also calls *exact* verification, as imposing a requirement of *holistic relevance*: for a fact to verify a given proposition, it must not contain any part which is irrelevant to the truth of the proposition (cf. his 2012a: p. 234 and 2014: p. 551f et passim).

[13] The non-distributive notion of verification has also been put to use in (Litland, 2016b) in developing a logic of a many-many notion of grounding, on which what is grounded is irreducibly a collection of truths or facts.

[14] The case of conjunction makes especially clear why the 'full' interpretation of 'by' must be assumed. For suppose that every fact s that verifies P also verifies Q and therefore $P \wedge Q$. By ordinary standards, it might then be true to say that every verifier s of P verifies $P \wedge Q$ by (among other things) verifying P. But it would not therefore be true to say that $P < P \wedge Q$. We avoid this result if we read 'by' as requiring fullness. For on such a reading, it is false that in general every verifier of P verifies $P \wedge Q$ by verifying P, since verifying P is usually only part of what is required for verifying $P \wedge Q$.

is falsified by exactly those facts that verify Q. For instance, the fact t' that Bob is 5 foot tall would plausibly falsify the proposition Q that Bob is tall, and hence verify $\neg Q$, whereas the fact t that Bob is 6 foot tall verifies Q, and therefore falsifies $\neg Q$.

We use again the $+/-$-notation to talk about bilateral contents and their components. That is, for P a bilateral content, P^+ is the positive component of P, and P^- is the negative component, so that $P = \langle P^+, P^- \rangle$. The notion of verification is applied with harmless ambiguity to both bilateral contents and their (unilateral) positive and negative components. Verification of a bilateral content P is defined as verification of P^+, and falsification of P as verification of P^-.

In giving an account of negation, I now need to say *how* a negation is verified by its verifiers, and falsified by its falsifiers. For each of these questions, two views are prima facie possible. I call them the *by-view* and the *identity-view*, respectively. So we have to consider four views with respect to $\neg P$:

(BV): The by-view of verification: $\neg P$ is verified by falsifying P.
(BF): The by-view of falsification: $\neg P$ is falsified by verifying P.
(IV): The identity-view of verification: $\neg P$ is verified how P is falsified.
(IF): The identity-view of falsification: $\neg P$ is falsified how P is verified.

For example, according to (BV), the fact t' would verify the proposition $\neg Q$ that Bob is not tall by falsifying its negatum Q, the proposition that Bob is tall. More generally, any fact verifying Q^-, by verifying Q^-, verifies $(\neg Q)^+$. And according to (BF), the state t that Bob is 6 foot tall falsifies $\neg Q$ by verifying Q. More generally, any verifier of Q^+, by verifying Q^+, verifies $(\neg Q)^-$. According to (IV), in contrast, $\neg Q$ is verified in exactly the ways Q is falsified, so that $(\neg Q)^+ = Q^-$. And on (IF), $\neg Q$ is falsified in exactly the ways Q is verified, so $(\neg Q)^- = Q^+$.

In principle, any view of verification can be consistently combined with any view of falsification, so we obtain a total of four possible views of how the bilateral content of a negation is determined: (BV+BF), (BV+IF), (IV+BF), and (IV+IF). Some of the views do not fit our desired principles for ground, though. Firstly, if we are to obtain that $P < \neg\neg P$, then it must hold that the double negation $\neg\neg P$ of a given content P may be verified *by* verifying P. But this rules out (IV+IF), since that view implies $P = \neg\neg P$. (IV+BF) and (BV+IF), in contrast, directly imply this result. For assume that s verifies P^+. Suppose we accept (BV+IF). By (IF), $(\neg P)^- = P^+$, so s verifies $(\neg P)^-$ in whatever ways it verifies P^+. By (BV), it follows that s verifies $(\neg\neg P)^+$ by verifying $(\neg P)^- = P^+$. But that just means that s verifies $\neg\neg P$ by verifying P, as desired. Now consider (IV+BF). By (BF), s verifies $(\neg P)^-$ by verifying P^+. By (IV),

$(\neg\neg P)^+ = (\neg P)^-$, so it follows that s verifies $(\neg\neg P)^+$ by verifying P^+, and thus s verifies $\neg\neg P$ by verifying P, as desired.

The situation is more complicated in the case of (BV+BF). Assume s verifies P^+. Then by (BF), s verifies $(\neg P)^-$ by verifying P^+. Moreover, by (BV), s verifies $(\neg\neg P)^+$ by verifying $(\neg P)^-$. Now, nothing we have said so far allows us to conclude that s verifies $(\neg\neg P)^+$ by verifying P^+. Nevertheless, it seems plausible that 'by', at least in its pertinent 'full' use, is *transitive* in the sense that if s φs by ψing, and ψs by χing, then s φs by χing. If so, then we may still infer that s verifies $(\neg\neg P)^+$ by verifying P^+. So it seems that all three remaining views will yield the desired mode of verification for the double negation $\neg\neg P$, and correspondingly that $P < \neg\neg P$.

The differences between the views emerge more clearly with respect to the other kinds of negative propositions, namely negations of conjunctions and disjunctions. Recall the above introduction principles for ground governing these:

($<$I$\neg\wedge$): If $\neg P$ then $\neg P < \neg(P \wedge Q)$ and $\neg P < \neg(Q \wedge P)$
($<$I$\neg\vee$): If $\neg P, \neg Q$, then $\neg P, \neg Q < \neg(P \vee Q)$

If we are to accommodate them, we must have that $\neg(P \wedge Q)$ may be verified by verifying $\neg P$ and by verifying $\neg Q$, and that $\neg(P \vee Q)$ may be verified by verifying $\neg P$, $\neg Q$. It turns out that this demand favours (IV+BF), the combination of the identity-view of the verification of a negation with the by-view of its falsification.

To see this, we first need to look at how conjunctions and disjunctions are falsified. Note that on the truthmaker conception, there is a strong analogy between the falsification of conjunctions and the verification of disjunctions, as well as between the falsification of disjunctions and the verification of conjunctions. Specifically, the negative content of a conjunction relates to the negative contents of its conjuncts like the positive content of a disjunction relates to the positive contents of its disjuncts. And similarly, the negative content of a disjunction relates to the negative contents of its disjuncts like the positive content of a conjunction relates to the positive contents of its conjuncts. It is natural to continue the analogy from the case of *what* verifies or falsifies to *how* it does so. In particular, just as $(P \vee Q)^+$ is verified via P^+ and via Q^+, we shall take $(P \wedge Q)^-$ to be verified via P^- and via Q^-, and just as $(P \wedge Q)^+$ is verified via P^+, Q^+, we shall take $(P \vee Q)^-$ to be verified via P^-, Q^-.

Now consider the claim that every verifier of $\neg P$, by verifying $\neg P$, verifies $\neg(P \wedge Q)$. First, we show that (IV+BF) implies this claim. Assume that s verifies $\neg P$. Then s verifies P^-, and so verifies $(P \wedge Q)^-$ by verifying P^-. By (IV), $(\neg(P \wedge Q))^+ = (P \wedge Q)^-$, so it follows that s verifies $(\neg(P \wedge Q))^+$ by verifying P^-. Moreover, we have $P^- = (\neg P)^+$, so it follows that s verifies $(\neg(P \wedge Q))^+$ by verifying $(\neg P)^+$, as desired. But now consider (BV+IF). If s verifies $(\neg P)^+$, then s verifies P^-, so s verifies $(P \wedge Q)^-$ via P^-. Given (BV), we have that s verifies $(\neg(P \wedge Q))^+$ via $(P \wedge Q)^-$. Assuming transitivity, we could infer that s verifies $(\neg(P \wedge Q))^+$ via P^-, but without the identity of P^- and $(\neg P)^+$, there is no way we can then obtain the conclusion that s verifies $(\neg(P \wedge Q))^+$ via $(\neg P)^+$. Moreover, as this case does not involve the falsification of any negation, the situation is exactly the same for (BV+BF).

To validate all of the desired introduction principles for ground, we therefore have to accept (IV+BF) as our account of negation. This account is striking in embodying an *asymmetric* view of negation. It distinguishes between the falsification of a negation and the verification of the negatum, but *not* between the verification of a negation and the falsification of the negatum. In other words, the view is that to falsify a content *is* to verify its negation, whereas it is not the case that to verify a content is to falsify its negation. Rather, falsifying the negation is something achieved through, but distinct from, the verification of the negatum.[15]

5. A MODE-IFIED TRUTHMAKER THEORY OF CONTENT

On the view I have proposed, the features of a truth to which ground is sensitive are features concerning what facts verify the truth and *how*, i.e. in what *modes* they do so. We therefore have to encode in (a mathematical representation of) a proposition both what facts verify or falsify it as well as in what modes they do so. The obvious idea is to replace the sets of facts in Fine's account by sets of *pairs* $\langle s, m\rangle$ of a fact s and a mode

[15] That this kind of asymmetric account should be accepted is a surprising result; independently of the connection to the introduction principles, it might have seemed more natural to endorse one of the symmetric accounts. So the question arises whether there might be independent philosophical reasons for endorsing the asymmetric account. Although the matter calls for a much more extended discussion than I can offer here, it may be worth mentioning one possible source of independent motivation. I have in mind the kind of asymmetric account of truth and falsity that is endorsed, for example, by Williamson (1994: p. 188), which can be captured by the following principles:

(T): If a proposition says that P, then it is true iff P.
(F): If a proposition says that P, then it is false iff $\neg P$.

This account immediately ties both the truth of a proposition saying that $\neg P$ and the falsity of a proposition saying that P to the same thing: it being the case that $\neg P$. But it does not in the same way tie the falsity of a proposition saying that $\neg P$ and the truth of a proposition saying that P to the same thing. Rather, the first is tied, in the first instance, to it being the case that $\neg\neg P$, and the second to it being the case that P. This asymmetry is at least strongly reminiscent of the identification of the falsification of P with the verification of $\neg P$, in the absence of the identification of the verification of P with the falsification of $\neg P$. As such, it may perhaps provide an independent basis for the latter.

m. The presence of $\langle s, m\rangle$ in P^+ (P^-) would then be taken to represent that s verifies (falsifies) P in mode m. The limiting case of a state verifying a proposition *directly*, as it were, i.e. not by verifying any propositions, may be represented by means of a special mode m_0 of directness. Any indirect mode m would be identified by the propositions $P_1, P_2, \ldots$ in the verification of which it consists. The truths $P_1, P_2, \ldots$ would be taken to ground a truth Q just in case for every fact s verifying $P_1, P_2, \ldots$, when m is the mode corresponding to $P_1, P_2, \ldots$, $\langle s, m\rangle$ is a member of Q.

I shall however deviate from this proposal in two ways. Firstly, it turns out that given our account of ground, we can work with a significantly simpler construction than the one just described: we may replace Fine's sets of facts simply with sets of modes, counting facts as special, *direct* modes of verification.[16] The presence of a fact s in P^+ represents that s verifies P directly, and the presence of an (indirect) mode m in P^+ represents that P is verified in mode m, by every fact verifying the propositions $Q_1, Q_2, \ldots$ corresponding to m. We may then take the propositions $P_1, P_2, \ldots$ to ground a proposition Q just in case there is a mode of verification corresponding to $P_1, P_2, \ldots$, and it is a member of Q. It is straightforward to show that the relation of ground holds according to the latter, simpler picture just in case it holds between the corresponding contents on the former, more complicated picture.

The second deviation is motivated by a desire to accommodate a *non-factive* understanding of ground. Roughly speaking, some propositions $P_1, P_2, \ldots$ non-factively ground a proposition Q iff they satisfy the conditions to ground Q, bar perhaps the condition of being true. This means in particular that non-factive ground satisfies unconditional versions of the introduction principles stated above, so that for example $P < P \vee Q$ holds irrespective of whether P or Q are true.[17] To characterize non-factive ground, we move to a similarly non-factive conception of indirect modes of verification.[18] Mainly,

16 The possibility of this simplification was suggested to me by [removed for blind review].

17 On the idea of a non-factive notion of ground, cf. e.g. (Fine, 2012b: 48ff), and (Correia, 2014b: p. 36).

18 A similar move may be considered for the direct modes. Specifically, we may follow Fine (cf. 2014: 557f; 2016: p. 8), and replace the appeal to the notion of a fact by an appeal to a broader notion of a state, which is like that of a fact except in that a state need not be actual, and indeed need not even be possible. There is then no obstacle to assuming every proposition to have at least one verifier and at least one falsifier. However, since ground, on my account, is determined purely by the presence or absence of indirect modes in a given proposition, this extension of the conception of verifiers is not required for our construction to work as intended. To the extent that non-actual and impossible modes are less problematic than non-actual and impossible states, it is an advantage of my framework that it can accommodate non-factive ground using less worrisome resources than are required on Fine's approach. (It should be noted, though, that if we do not allow non-actual and impossible states, then

this means that we do not demand that the propositions $P_1, P_2, \ldots$ be true if they are to correspond to a mode of verification. We may then include in $P \vee Q$ the mode of verifying via P, irrespective of whether P is true. We can then say that P non-factively grounds $P \vee Q$ on the strength of the fact that $P \vee Q$ contains the mode of verifying via P. Given a suitable selection of modes as *actual*, we may then count a proposition true just in case one of its modes is actual, and factive ground may be defined in the natural way in terms of truth and non-factive ground. For simplicity, I shall henceforth focus exclusively on non-factive ground.

Our first task will be to describe the basic behaviour of modes, this is done in 5.1. Section 5.2 defines notions of ground on our contents, and in 5.3 I then define suitable notions of conjunction, disjunction, and negation, which are shown to relate to ground in the desired way. Section 5.4 identifies the constraints on our ground-theoretic contents corresponding to the structural features ground is sometimes taken to possess. In 5.5 I turn to the matter of the identity conditions on modes, and explain how they bear on the question of equivalence in ground-theoretic content.

5.1. **Modes.** We shall begin by describing the basic behaviour of modes, leaving open their exact nature and identity.

Firstly, any mode is either *direct*, which is to say that it is a fact, or it is *indirect*, which is to say that there is some list of propositions $P_1, P_2, \ldots$ such that m is the mode of verifying *via* $P_1, P_2, \ldots$. We will also call direct modes *fundamental*, and indirect ones *derivative*. Given our informal account of the previous section, it is clear that *some* lists of propositions determine a mode, i.e. that there are some derivative modes. For instance, the verifiers of the conjunction $P \wedge Q$ of truths P, Q verify the conjunction by verifying P, Q. So there is a mode of verification corresponding to the list of propositions P, Q.

There appears to be no motivation, intuitive or theoretical, for allowing distinct modes to correspond to the same list of propositions. The mathematical structure corresponding to a list is a sequence, so we may take there to be a function mapping certain sequences of propositions to modes. We shall write this function V, alluding to the fact that the mode in question will be the mode of verifing *via* the relevant list of propositions. So when $\langle P_1, P_2, \ldots \rangle$ is a suitable sequence of propositions, $V\langle P_1, P_2, \ldots \rangle$ is the mode of verifying via $P_1, P_2, \ldots$.

the presence of a mode m in a proposition P does not in general represent exactly that every verifier of the propositions corresponding to m thereby verifies P. For the latter condition will be vacuously satisfied for any non-actual mode. The presence of m in P should then simply be understood to represent that verifying the propositions corresponding to m is a way to verify P – though it may be logically impossible to verify P in this way.)

The basic structure that we will work with is a *mode-space*, which is a pair $\langle M, V\rangle$ of a non-empty set of modes M, and a via-function V. A maximally liberal conception of propositions is obtained as follows. We call any subset P of M a *unilateral proposition*, which is verified in exactly the modes which are its members. And we call any pair $\mathbf{P} = \langle \mathbf{P}^+, \mathbf{P}^-\rangle$ of unilateral propositions a *bilateral proposition*, which is verified in exactly the modes that are members of $\mathbf{P}^+$, and falsified in exactly the modes that are members of $\mathbf{P}^-$. (Here and in what follows, I mark the unilateral/bilateral distinction typographically by using boldface variables for bilateral propositions.) For now, we may focus on unilateral propositions.

The function V is assumed to be a mapping from some set of sequences of propositions into M. It is not required that V be defined for every sequence of propositions, or even that it be defined for every singleton sequence $\langle P\rangle$ with P a proposition. Informally, that V is defined for some sequence of propositions $\langle P_1, P_2, \ldots\rangle$ means that there is such a thing as doing something by verifying $P_1, P_2, \ldots$ Correspondingly, if V is undefined for $\langle P_1, P_2, \ldots\rangle$, this means that there is no such thing as doing something by verifying $P_1, P_2, \ldots$ One reason for not requiring that V be defined for all sequences of propositions is that such a requirement would be inconsistent with a view of ground as irreflexive. To see this, note that M is itself a proposition, so $V\langle P\rangle$ should have to be defined for $P = M$. But then $V\langle P\rangle$ would have to a member of P, which is to say that verifying P is a way to verify P. Given the proposed account of ground in terms of modes of verification, it would follow that P grounds itself.

We shall however take for granted that two natural closure principles hold for the range of sequences on which V is defined. Firstly, if V is defined for a given sequence $\langle P_1, P_2, \ldots\rangle$ then it is also defined for any non-empty subsequence. Secondly, if V is defined for each of the sequences $\gamma_1, \gamma_2, \ldots$, then V is also defined for their *concatenation* $\gamma_1{}^\frown\gamma_2{}^\frown\ldots$.[19] For simplicity, we require that V be undefined for the empty sequence $\langle\rangle$.[20]

19 For present purposes, we may restrict attention to concatenations of an at most countable sequence of sequences.

20 There may be purposes for which a putative 'nullmode' corresponding to $\langle\rangle$ may be useful. If a mode is counted actual iff all propositions in the corresponding sequence are true, then the nullmode would automatically be actual and hence every proposition containing it trivially true. It would thereby have a similar profile to the *nullfact* (or nullstate) in Fine's framework, which is the fusion of the empty set of facts (or states), and part of every fact (state). The most obvious application of the nullmode would be to capture Fine's idea that some truths may be *zero-grounded*, where this is supposed to be distinct from being ungrounded; cf. (Fine, 2012b: p. 47f).

Note that V is not required to be one-to-one, but is allowed to map different sequences to the same mode. We shall assume that the property of determining the same mode is preserved under concatenation of sequences. That is, if $V(\gamma_1) = V(\delta_1)$, $V(\gamma_2) = V(\delta_2)$, ..., then $V(\gamma_1{}^\frown\gamma_2{}^\frown\ldots) = V(\delta_1{}^\frown\delta_2{}^\frown\ldots)$. If $m_1 = V(\gamma_1)$, $m_2 = V(\gamma_2), \ldots$, we call $V(\gamma_1{}^\frown\gamma_2{}^\frown\ldots)$ *a fusion of* $\langle m_1, m_2, \ldots\rangle$. Given the previous constraints, any sequence of indirect modes $\langle m_1, m_2, \ldots\rangle$ has a unique fusion which we denote by $\bigsqcup\langle m_1, m_2, \ldots\rangle$ or $m_1 \sqcup m_2 \sqcup \ldots$

Whenever $m = V\langle P_1, P_2, \ldots\rangle$, we call the *set* $\{P_1, P_2, \ldots\}$ a *ground-set* of m, since $\{P_1, P_2, \ldots\}$ will ground a proposition Q if $m \in Q$. A mode-space will be called *constrained* iff: if V maps two sequences $\langle P_1, P_2, \ldots\rangle$ and $\langle Q_1, Q_2, \ldots\rangle$ of propositions to the same mode, then the corresponding sets $\{P_1, P_2, \ldots\}$ and $\{Q_1, Q_2, \ldots\}$ are identical. In that case, any indirect mode m has a unique ground-set, which we denote by $|m|$. We shall usually assume that we are working in a constrained mode-space.[21]

Having described what modes are like, it is natural to ask what modes *are*. In view of their intimate relationship with sequences of propositions, an obvious suggestion is that perhaps modes may simply be *identified* with these sequences. With respect to a large variety of mode-spaces, such an identification may indeed be carried through. There are, however, interesting sorts of mode-spaces for which this is not possible. The basic point is that sequences of propositions are set-theoretic constructions from propositions, and propositions themselves are set-theoretic constructions from modes. As a result, if modes are identified with sequences of propositions, the relation of ground will automatically inherit certain features of the membership-relation, in particular its irreflexivity, its asymmetry, and its well-foundedness. So for any mode-spaces that yield relationships of ground that violate these principles, the reduction of modes to sequences, or other set-theoretic constructions, of propositions, cannot be carried out. Whether a different useful kind of reduction is then possible, perhaps using non-well-founded sets, is a question I shall leave for another occasion. At any rate, in devising a framework for theorizing about ground, I believe we are well-advised first to try and work out what sort of constraints different plausible theories of ground impose on the modes and their structure. Once we understand this, we may return to the metaphysical question of what sorts of things modes may be taken to be on the various views. For the time being, we shall therefore adopt towards modes the stance that Fine adopts towards facts or states, and simply take them as given.

21 It may be worth pointing out that for cardinality reasons, in a constrained mode-space, V will be undefined for most sequences. Thanks here to an anonymous referee.

5.2. **Ground.** We first define notions of (non-factive) ground for unilateral contents. Given our informal account of strict full ground above, we may define $<$ and its partial cousin $\prec$ as follows. Let Γ be a non-empty set of (unilateral) contents and P and Q (unilateral) contents. Then:

($<$): $\Gamma < P$ iff for some mode $m \in P$, Γ is a ground-set of m
($\prec$): $P \prec Q$ iff $\Delta, P < Q$ for some set of propositions Δ

Note that in contrast to Fine's account above, strict ground is given a direct definition in terms of modes of verification, and not defined via a reflexive notion of weak ground. Nevertheless, we shall later have use for such a notion, and for a partial version of it, which we define as follows:

($\leq$): $\Gamma \leq P$ iff $\Gamma < P$ or $\Gamma = \{P\}$ or ($\{P\} \subset \Gamma$ and $\Gamma \backslash \{P\} < P$)
($\preceq$): $P \preceq Q$ iff $P = Q$ or $P \prec Q$

The following principles are straightforward consequences from these definitions:[22]

Identity($\leq$): $P \leq P$.
Subsumption($<$/$\prec$): If $\Gamma, P < Q$ then $P \prec Q$.
Subsumption($<$/$\leq$): If $\Gamma < Q$ then $\Gamma \leq Q$.
Subsumption($\leq$/$\preceq$): If $\Gamma, P \leq Q$ then $P \preceq Q$.
Subsumption($\prec$/$\preceq$): If $P \prec Q$ then $P \preceq Q$.

The only non-trivial case is Sub($\leq$/$\preceq$). But suppose $\Gamma, P \leq Q$. There are three cases. (i) $\Gamma, P < Q$. Then $P \prec Q$ and hence $P \preceq Q$. (ii) $\Gamma \cup \{P\} = \{Q\}$. Then $P = Q$ and hence $P \preceq Q$. (iii) $\{Q\} \subset \Gamma \cup \{P\}$ and $(\Gamma \cup \{P\}) \backslash \{Q\} < Q$. Then either $P = Q$ and hence again $P \preceq Q$, or $P \in (\Gamma \cup \{P\}) \backslash \{Q\}$, and hence $P \prec Q$, and so $P \preceq Q$.

We might also have defined weak partial ground as the partial version of weak full ground rather than, as we have done above, as a weak version of strict partial ground. For given the other definitions, the condition that $P \prec Q$ or $P = Q$ is equivalent to the condition that for some Δ, we have $\Delta, P \leq Q$. The right-to-left direction was just established. For the left-to-right direction, suppose first that $P \prec Q$. Then for some Δ, we have $\Delta, P < Q$ and hence $\Delta, P < Q$. Suppose then that $P = Q$. Then $P \leq Q$, and hence again, for some Δ, we have $\Delta, P \leq Q$.

The extension of the four notions of ground to bilateral contents is done in the simplest possible way. For a non-empty set $\mathbf{\Gamma}$ of bilateral contents, let $\mathbf{\Gamma}^+ = \{\mathbf{P}^+ : \mathbf{P} \in \Gamma\}$. Then we define $\mathbf{\Gamma} < \mathbf{P}$ by $\mathbf{\Gamma}^+ < \mathbf{P}^+$, and likewise in the other cases.

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22 I borrow the labels from the corresponding inference rules in Fine (2012c)'s pure logic of ground. – Here and in what follows, I adopt the familiar convention of writing $\Gamma \cup \{Q\}$ as Γ, Q as well as $\Gamma \cup \Delta$ as Γ, Δ, and similarly in other cases.

5.3. **The Truth-Functional Operations.** In this section I define truth-functional operations on our ground-theoretic contents. They are shown to combine with the above account of ground to yield an attractive account of the grounds of truth-functionally complex contents, which includes the unrestricted introduction principles for ground. We first define conjunction and disjunction for unilateral contents. We then extend these operations to the case of bilateral contents and define an operation of negation. We then establish general necessary and sufficient conditions for some propositions to ground the various kinds of truth-functionally complex propositions.

Recall the informal account given in the previous section of how disjunctions, conjunctions, and negations are verified. With respect to disjunction, we said that any verifier of P verifies $P \vee Q$ by verifying P, and any verifier of Q verifies $P \vee Q$ by verifying Q. And with respect to conjunction, we said that any fusion $s \sqcup t$ of verifiers s of P and t of Q verifies $P \wedge Q$ by verifying P, Q. It follows that given any disjunction $P \vee Q$, V must be defined for $\langle P \rangle$, and $\langle Q \rangle$, and $P \vee Q$ must include both $V\langle P \rangle$ and $V\langle Q \rangle$. Likewise for any conjunction $P \wedge Q$, V must be defined for $\langle P, Q \rangle$, and $P \wedge Q$ must include $V\langle P, Q \rangle$.

Plausibly, both conjunctions and disjunctions may also be verified in other ways. Thus, suppose that P may be verified by verifying some propositions $R_1, R_2, \ldots$ Then it is natural to hold that $P \wedge Q$ may also be verified by verifying $R_1, R_2, \ldots, Q$. Similarly, if Q may be verified by verifying $R_1, R_2, \ldots$, it is natural to say that $P \wedge Q$ may also be verified by verifying $P, R_1, R_2, \ldots$ And finally, if P and Q may be verified via $R_1, R_2, \ldots$ and $S_1, S_2, \ldots$, respectively, $P \wedge Q$ may be verified via $R_1, R_2, \ldots, S_1, S_2, \ldots$

In the case of the disjunction $P \vee Q$, it seems similarly plausible that it may be verified not only via P and via Q, but also via $R_1, R_2, \ldots$ whenever either of P and Q may be so verified. In addition, we shall suppose that every mode of verifying the conjunction $P \wedge Q$ is also automatically a mode of verifying $P \vee Q$.[23]

We may usefully state this account of conjunction and disjunction more formally in terms of three auxiliary operations on unilateral propositions. At certain key places, we need to ensure that V is defined for $\langle P \rangle$. We shall say that a proposition P is *raisable* exactly when this is the case. Then when P, Q are propositions including only derivative modes, and when R is a raisable proposition:

(Df. $\sqcup$): $P \sqcup Q := \{m \sqcup n : m \in P \text{ and } n \in Q\}$
(Df. $+$): $P + Q := (P \cup Q) \cup (P \sqcup Q)$
(Df. $\Uparrow$): $\uparrow R := \{V\langle R \rangle\} + \{m \in R : m \text{ is derivative}\}$

23 This parallels Fine's account of the truthmakers of disjunctions which include not only the verifiers of the disjuncts, but also any fusions of these.

Note that there is a close correspondence between the operations $\sqcup$ and $+$ on unilateral contents and Fine's operations of conjunction and disjunction, respectively. For the way $P \sqcup Q$ is obtained from P and Q is exactly analogous to how, one Fine's view, the positive content of a conjunction is obtained from that of its conjuncts. Likewise the way $P+Q$ is obtained from P and Q is analogous to how, one Fine's view, the positive content of a disjunction is obtained from that of its disjuncts. On the present account, however, conjunction and disjunction are defined in terms of these operations and the third one, which I call *raising*. Its effect is to produce a proposition $\uparrow P$ just like its argument P, except that it may be verified via P, and via $P, R_1, R_2, \ldots$ whenever P may be verified via $R_1, R_2, \ldots$[24] The following definitions of unilateral conjunction and disjunction accord with our above account of what modes should be included in $P \vee Q$ and $P \wedge Q$. For raisable propositions P, Q:

(Df. $\wedge$U): $P \wedge Q := \uparrow P \sqcup \uparrow Q$
(Df. $\vee$U): $P \vee Q := \uparrow P + \uparrow Q$

Given our informal discussion of negation in the previous section, it is immediate how conjunction, disjunction, and negation are now to be defined on bilateral contents. For pairs of raisable unilateral propositions $\mathbf{P}$, $\mathbf{Q}$:

(Df. $\neg$B): $\neg \mathbf{P} := \langle \mathbf{P}^-, \uparrow \mathbf{P}^+ \rangle$
(Df. $\wedge$B): $\mathbf{P} \wedge \mathbf{Q} := \langle \mathbf{P}^+ \wedge \mathbf{Q}^+,\ \mathbf{P}^- \vee \mathbf{Q}^- \rangle$
(Df. $\vee$B): $\mathbf{P} \vee \mathbf{Q} := \langle \mathbf{P}^+ \vee \mathbf{Q}^+,\ \mathbf{P}^- \wedge \mathbf{Q}^- \rangle$

Note how the definition of $\neg$ reflects the *asymmetric* view of negation.

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24 Note that absent any assumptions to the effect that P cannot be verified in part by verifying P, there is no guarantee that $\uparrow P \neq P$. – One might wonder whether an argument is not needed for the claim that there always exists such a proposition as $\uparrow P$. Formally, the assumption that P is raisable, in conjunction with the closure of the set of modes under fusion, makes sure that a suitable proposition always exists. But we may then ask for a defence of this assumption; what justifies disregarding propositions that are not raisable? The simplest answer is perhaps this. For any legitimate proposition $\mathbf{P}$, it should be possible to form its double negation $\neg\neg\mathbf{P}$. Given the proposed account of ground and negation, $(\neg\neg\mathbf{P})^+$ relates to $\mathbf{P}^+$ exactly so that $\uparrow(\mathbf{P}^+) = (\neg\neg\mathbf{P})^+$ (similarly for $(\neg\neg\mathbf{P})^-$). So whenever $\uparrow P$ does not exist, P cannot occur within a legitimate bilateral content and may for that reason be discarded. We can perhaps also argue for the raisability of legitimate propositions on independent grounds. For it seems that if P is a legitimate (unilateral) proposition, then there is such a thing as (non-factively) verifying P – if no sense can be made of the idea of P being verified, there is something incoherent about P. But if there is such a thing as verifying P, then it seems we can ask what can be done *by* verifying P. Now this question is about a way, or mode of doing something, namely the mode of doing it by verifying P. So this mode should be taken to exist. But since this mode is just the mode $V\langle P \rangle$, it follows that P is raisable.

We may then establish substantive necessary and sufficient conditions for a set of propositions $\mathbf{\Gamma}$ to ground any kind of truth-functionally complex proposition. To state them concisely, we introduce some abbreviations. Let $\mathbf{\Gamma} \leq \{\mathbf{P}_1, \mathbf{P}_2, \ldots\}$ abbreviate: for some sets $\mathbf{\Gamma}_1, \mathbf{\Gamma}_2, \ldots$ with $\mathbf{\Gamma} = \mathbf{\Gamma}_1 \cup \mathbf{\Gamma}_2 \cup \ldots$, $\mathbf{\Gamma}_1 \leq \mathbf{P}_1$ and $\mathbf{\Gamma}_2 \leq \mathbf{P}_2$ and .... Then for raisable propositions $\mathbf{P}$ and $\mathbf{Q}$ and any set of raisable propositions $\mathbf{\Gamma}$:[25]

($< \wedge$): $\mathbf{\Gamma} < \mathbf{P} \wedge \mathbf{Q}$ iff $\mathbf{\Gamma} \leq \{\mathbf{P}, \mathbf{Q}\}$
($< \vee$): $\mathbf{\Gamma} < \mathbf{P} \vee \mathbf{Q}$ iff $\mathbf{\Gamma} \leq \mathbf{P}$ or $\mathbf{\Gamma} \leq \mathbf{Q}$ or $\mathbf{\Gamma} \leq \{\mathbf{P}, \mathbf{Q}\}$
($< \neg\neg$): $\mathbf{\Gamma} < \neg\neg\mathbf{P}$ iff $\mathbf{\Gamma} \leq \mathbf{P}$
($< \neg\wedge$): $\mathbf{\Gamma} < \neg(\mathbf{P} \wedge \mathbf{Q})$ iff $\mathbf{\Gamma} \leq \neg\mathbf{P}$ or $\mathbf{\Gamma} \leq \neg\mathbf{Q}$ or $\mathbf{\Gamma} \leq \{\neg\mathbf{P}, \neg\mathbf{Q}\}$
($< \neg\vee$): $\mathbf{\Gamma} < \neg(\mathbf{P} \vee \mathbf{Q})$ iff $\mathbf{\Gamma} \leq \{\neg\mathbf{P}, \neg\mathbf{Q}\}$

By the reflexivity of $\leq$, the introduction principles for ground can be obtained from the right-to-left directions of these biconditionals. For example, since $\mathbf{P} \leq \mathbf{P}$ and $\mathbf{Q} \leq \mathbf{Q}$, it follows that $\mathbf{P}, \mathbf{Q} \leq \{\mathbf{P}, \mathbf{Q}\}$, and hence by the right-to-left direction of ($< \wedge$), $\mathbf{P}, \mathbf{Q} < \mathbf{P} \wedge \mathbf{Q}$. The left-to-right directions of the biconditionals in turn correspond exactly to the elimination rules for ground proposed by Fine (2012b: p. 63ff).[26]

A number of variations on the above definitions may be considered. For example, we might not allow (unilateral) disjunctions $P \vee Q$ to be verified via P, Q, but only via P and via Q, as well as perhaps in modes in which the latter are verified. The above results would then still hold once we drop the third disjunct from the right-hand sides of ($< \vee$) and ($< \neg\wedge$). It would also be very interesting to see exactly what logical principles would hold on a symmetric account of negation.

What about views, like that proposed in (Correia, 2010), which impose substantive restrictions even on, say, the principle that $P < P \vee Q$? Can they also be accommodated within the present framework? There is no general reason why this should not be possible. As long as the restrictions imposed can somehow be captured in terms of the modes in which the component propositions are verified, we may simply exclude the offending

25 A proof of this result is given in the appendix.

26 The elimination rules are perhaps more controversial than the introduction rules; for instance, it might be suggested that $P \vee \neg P$ is not only grounded by the weak grounds of its true disjunct, but also by the laws of logic, on some suitable construal of that phrase (this idea is also mentioned, but not endorsed in (Schnieder, 2011: p. 457f)). So it may be worth noting that the elimination rules may be invalidated in a natural way without the introduction rules thereby also becoming invalid. All we need to do is to drop the requirement that the mode-space be constrained. (Whether and how the suggestion that the laws of logic ground $P \vee \neg P$ could be implemented within the overall framework developed here is harder to answer, and depends strongly on how exactly that view is spelt out.) Thanks here to an anonymous referee.

modes from the complex propositions as defined above. We might, for example, stipulate that $P \vee Q$ is verified via $V\langle P\rangle$ and $V\langle Q\rangle$ only if $P \neq Q$. This would ensure that $P \not< P \vee P$.[27]

Note, however, that these sorts of moves cannot be motivated in the way in which Correia argues for his favoured restrictions on the introduction principles. For as we saw earlier, Correia appeals to a *prior* standard for the individuation of ground-theoretic content on which P and $P \vee P$ are identified. But when ground-theoretic content is conceived as on the present view as capturing in what modes a content is verified, then the latter question needs to be answered independently of assumptions concerning the identity and distinctness of the contents involved. I take this to provide some additional support for Correia's contention that the view of ground as satisfying the unrestricted introduction principles targets a *different notion* of ground, sensitive to different features of a sentence's overall content, than the view on which only a restricted version of the principles hold (cf. Correia's 2010: pp. 256ff; 2014a: §§3, 5; 2014b: p. 36).

5.4. Structural Properties of Ground.

In this section we discuss some structural properties that ground may be taken to possess, and how they may be captured within my framework. There is in principle an unsurveyable number of such properties; my choice in which of them to discuss has been guided in part by which look particularly natural from the perspective of my framework, and by which of them are endorsed in the currently most well-developed view on the matter, which is that of Fine (2012c).

It is often held that ground is irreflexive in the sense that nothing helps ground itself.[28] Given our definition of ground, this amounts to the claim that there is no proposition P such that P is a member of a ground-set of some mode $m \in P$.[29] Nothing we have said

27 It might also be possible to achieve the same result not by changing the definition of disjunction, but by revising the definition of grounding, giving up on the tight connection that every mode of verification corresponds to an instance of grounding. – Note that the wish to reject $P < P \vee P$ is not the only possible motivation for wanting to restrict the disjunction principle that $P < P \vee Q$; some of the possible responses to the puzzles of ground presented in (Fine, 2010) also involve such a restriction. With respect to these views, the same comments apply: as long as the relevant restrictions can be captured within our framework, the views can be accommodated. Thanks to an anonymous referee for bringing up the matter of the puzzles of ground.

28 Since this is the standard view, more interesting than listing sources for it is to give some sources where the principle has been called into question. One context in which self-grounding has been considered a possibility is that of the paradoxes of ground described in Fine (2010) and Krämer (2013); see in particular (Correia, 2014b: sec. 7). Different kinds of doubts about irreflexivity are raised in Jenkins (2011).

29 Throughout this section, by 'content' and 'proposition' I shall mean unilateral content.

so far guarantees this. Specifically, given a proposition P, nothing we have said rules out that V maps $\langle P\rangle$ to a member of P, in which case $P < P$.

Call a proposition P *irreflexive* iff P is not an element of any sequence γ with $V(\gamma) \in P$. If $V\langle P_1, P_2, \ldots\rangle \notin \bigcup\{P_1, P_2, \ldots\}$ for any propositions $P_1, P_2, \ldots$, then all propositions are irreflexive. For irreflexive propositions, and *only* for them, the following principles hold:[30]

Irreflexivity($<$): $\Gamma, P \not< P$.
Irreflexivity($\prec$): $P \not\prec P$.

Irreflexive propositions also satisfy the following principle.[31]

Reverse Subsumption($\leq$/$<$): If $\Gamma \leq P$ and $Q \prec P$ for all $Q \in \Gamma$, then $\Gamma < P$.

This principle corresponds to a basic rule of inference in Fine's logic; it is closely related to the characterization of strict full ground as 'irreversible' weak full ground, i.e. the equivalence of $\Gamma < P$ to $\Gamma < P$ and $P \not\preceq Q$ for any $Q \in \Gamma$. The right-to-left direction of this equivalence is ensured already by our definition of $\leq$. For suppose $\Gamma \leq P$ and $P \not\preceq Q$ whenever $Q \in \Gamma$. Unless $\Gamma < P$, then from the definition of $\leq$ it follows immediately that $P \in \Gamma$. But since $P \preceq P$, this contradicts the assumption that $P \not\preceq Q$ whenever $Q \in \Gamma$.

The left-to-right direction of the equivalence holds whenever another commonly accepted principle for ground holds, namely that ground is asymmetric in the sense that if P helps ground Q, Q does not help ground P. We may call a proposition P *asymmetric* iff whenever $V\langle Q_1, Q_2, \ldots\rangle \in P$ and $V\langle R_1, R_2, \ldots\rangle \in \bigcup\{Q_1, Q_2, \ldots\}$, then P is not among $R_1, R_2, \ldots$ It is straightforward to show that for exactly the asymmetric propositions P:

Asymmetry($<$): If $\Gamma, Q < P$, then $\Delta, P \not< Q$.
Asymmetry($\prec$): If $Q \prec P$, then $P \not\prec Q$.

But now suppose $\Gamma < P$. Then, firstly, $\Gamma \leq P$. Secondly, the relationship of weak ground is irreversible. For suppose $P \preceq Q$ for some $Q \in \Gamma$. Then either $P \prec Q$ or $P = Q$. But since $Q \prec P$, Asymmetry($\prec$) rules out both possibilities. So strict full ground implies irreversible weak full ground for irreflexive and asymmetric propositions.

I wish to briefly mention two further principles of interest related to those just considered.

Redundancy($\leq$): If $\Gamma, P \leq P$ then $\Gamma \leq P$.

[30] For the first principle, assume $\Gamma, P < P$. Then Γ, P is a ground-set of some derivative mode $m \in P$, so for some sequence of propositions γ with $V(\gamma) = m$, Γ, P is the set underlying γ. But then P is an element of γ, contrary to our assumption. The second principle follows by the definition of $\prec$. The other directions are equally straightforward.

[31] Assume $\Gamma \leq P$ and $Q \prec P$ for all $Q \in \Gamma$. Note that $P \notin \Gamma$, for otherwise $P \prec P$, contradicting irreflexivity. But then neither $\Gamma = \{P\}$ nor $\{P\} \subset \Gamma$, and therefore $\Gamma < P$.

Reverse Subsumption($\prec$/$=$): If $P \preceq Q$ and $Q \preceq P$, then $P = Q$.

The first one follows from irreflexivity, the second from asymmetry.[32]

The principles are interesting in part because they do *not* hold in the logic of (Fine, 2012c). To see this, let fact s be a proper part of fact t, i.e. $t = t \sqcup s$ and $t \neq s$. Then consider $P = \{s, t\}$ and $Q = \{t\}$. Evidently, $P \neq Q$. But $Q \subseteq P$, so on Fine's account, $Q \leq P$, and hence $Q \preceq P$. Moreover, since $s \sqcup t = t \sqcup t = t$, every fusion of verifiers of P and Q is a verifier of Q, so we have $P, Q \leq Q$, and hence $P \preceq Q$. So P and Q are a counter-example to Reverse Subsumption($\prec$/$=$), and indeed to the weaker claim that mutual weak *full* ground implies identity. Moreover, they also give rise to a counter-example to Redundancy($\leq$), for as we saw, $P, Q \leq Q$, whereas $P \not\leq Q$. The differences between Fine's approach to weak ground and ours thus manifest themselves in significant differences concerning the structural properties of weak ground.

It is also often maintained that ground is transitive.[33] A number of different principles may be subsumed under this heading. Relatively weak examples are that if $P \prec Q$ and $P \prec R$, then $P \prec R$, or that if $\Gamma < P$, and $P < Q$, then $\Gamma < Q$. A stronger form of transitivity may be stated as follows:

Transitivity($<$): If $\Gamma < P$ and $\Delta, P < Q$, then $\Gamma, \Delta < Q$.

This principle holds, for example, on the views of (Fine, 2012c: pp. 5, 22) and (Correia, 2010: p. 262).

Let us then consider what might be natural constraints of transitivity to impose on our contents. Here is one obvious idea, corresponding to the second weak interpretation of transitivity above. Suppose Q contains $V\langle P\rangle$, and P contains $V(\gamma)$. Then it might naturally be required that $V(\gamma)$ should also be contained in Q. This would ensure that if $\Gamma < P$, and $P < Q$, then $\Gamma < Q$.

But consider now the case in which Q does not contain $V\langle P\rangle$, but contains some mode $V(\delta)$ with P an element of δ. A natural suggestion is that transitivity requires that Q also contain $V(\delta^{\gamma/\langle P\rangle})$ where $\delta^{\gamma/\langle P\rangle}$ results from systematically replacing $\langle P\rangle$ in δ by γ.[34]

32 For the first principle, assume $\Gamma, P \leq P$ for non-empty Γ. By irreflexivity, $\Gamma, P \not< P$. There remain two cases. (i) $\Gamma \cup \{P\} = \{P\}$. Then $\Gamma = \{P\}$, and hence $\Gamma \leq P$. (ii) $(\Gamma \cup \{P\}) \setminus \{P\} \leq P$. If $P \in \Gamma$, then $\Gamma \cup \{P\} = \Gamma$ and hence by assumption $\Gamma \leq P$. If $P \notin \Gamma$, then $(\Gamma \cup \{P\}) \setminus \{P\} = \Gamma$ and hence again, $\Gamma \leq P$. The second principle is immediate given asymmetry and the definition $\preceq$.

33 Again, it may be more interesting to mention dissenters. The most influential arguments against transitivity are perhaps those given in (Schaffer, 2012: p. 126ff). For replies, see Litland (2013); Raven (2013); Krämer and Roski (2016).

34 We might perhaps also consider allowing the replacement of only some occurrences of $\langle P\rangle$, which would create some additional complications because what modes can be obtained by transitivity from a mode can then not be read off from the corresponding ground-set.

This, however, is not sufficient for Transitivity($<$). For let Γ be the set corresponding by γ, so $\Gamma < P$. Now suppose Q contains $V\langle P, R\rangle$ with $R \neq P$, so $P, R < Q$. Now let $\Delta = \{P, R\}$, and note that since $\Delta = \Delta$, $P < Q$. So by Transitivity($<$), $\Gamma, \Delta = \Gamma, P, R < Q$. However, the envisaged transitivity constraint on contents yields only that Q contains $V(\gamma^\frown\langle R\rangle)$. The ground-set of this mode is Γ, R which is distinct from Γ, P, R unless $P \in \Gamma$. So there is no guarantee that $\Gamma, P, R < Q$.

The transitivity principle for ground validated by this constraint may instead be stated thus:

Transitivity($<$)*: If $\Gamma < P$ and $\Delta < Q$, then $\Delta^{\Gamma/P} < Q$.

where $\Delta^{\Gamma/P}$ is $(\Delta\backslash\{P\}) \cup \Gamma$ if $P \in \Delta$ and Δ otherwise. If, on the other hand, we wish to ensure the stronger transitivity principle, we may impose the following constraint: If Q contains a mode with ground-set Δ, P, then Q contains a mode with ground-set Γ, Δ whenever P contains a mode with ground-set Γ.[35] Put in terms of corresponding via-sequences, we might say that if Q contains $V(\delta)$, P is an element of δ, and P contains $V(\gamma)$, then Q contains $V(\delta')$ whenever δ' may be obtained from δ by systematically replacing $\langle P\rangle$ in δ by either γ or $\langle P\rangle^\frown\gamma$. However, from an intuitive point of view, this seems to have a weaker claim than the previous constraint to amount simply to a principle of transitivity concerning modes of verification.

The final structural principle I wish to consider here is the following principle of amalgamation, which is derivable in Fine's pure logic of ground (cf. his 2012c: p. 7):

Amalgamation($<$): If $\Gamma_1 < P$ and $\Gamma_2 < P$ and $\ldots$, then $\Gamma_1 \cup \Gamma_2 \cup \ldots < P$.

Like the previous principles, it does not hold for arbitrary contents. We may say that a content is *closed* iff, whenever it contains modes with ground-sets $\Gamma_1, \Gamma_2, \ldots$, it also contains some mode with ground-set $\Gamma_1 \cup \Gamma_2 \cup \ldots$. Then Amalgamation($<$) holds exactly for closed propositions P.

Call a proposition *normal* iff it is closed, irreflexive, and satisfies the strong transitivity constraint. We may then report two welcome results about normal propositions.[36] Firstly, normal propositions satisfy all the principles corresponding to the rules of the pure logic of ground advocated in (Fine, 2012c: p. 5).[37] Most of them have already been stated, the remaining ones are principles of transitivity involving weak grounding relationships:

Transitivity($\leq$/$\leq$): If $\Gamma \leq P$ and $\Delta, P \leq Q$ then $\Gamma, \Delta \leq P$
Transitivity($\preceq$/$\prec$): If $P \preceq Q$ and $Q \prec R$ then $P \prec R$

35 It is immediate that Transitivity($<$) will hold exactly for the propositions satisfying this constraint.
36 The proofs are in the appendix.
37 As we have seen, however, they satisfy some additional principles as well.

Transitivity($\prec$/$\preceq$): If $P \prec Q$ and $Q \preceq R$ then $P \prec R$
Transitivity($\prec$/$\prec$): If $P \preceq Q$ and $Q \preceq R$ then $P \preceq R$

Secondly, the property of normality is preserved under our truth-functional operations. Our understanding of conjunction, disjunction, and negation therefore coheres with a view of ground as satisfying the normality properties in the sense that if we start with a range of normal propositions, application of our truth-functional operations to them will never take us to non-normal propositions.[38]

5.5. **Ground-Theoretic Equivalence and the Individuation of Modes.** We have so far said very little about the conditions under which contents obtained by application of truth-functional operations are identical. Closely related, but even more important for the purposes of the theory of ground is the question under what conditions they are *ground-theoretically equivalent*, where this term is understood in accordance with the following definition:[39]

($\approx$)**:** $\mathbf{P}$ is ground-theoretically equivalent to $\mathbf{Q}$ ($\mathbf{P} \approx \mathbf{Q}$) iff
(i) for all $\mathbf{\Gamma}$: $\mathbf{\Gamma} < \mathbf{P}$ iff $\mathbf{\Gamma} < \mathbf{Q}$, and
(ii) for all $\mathbf{\Delta}$ and $\mathbf{R}$: $\mathbf{\Delta}, \mathbf{P} < \mathbf{R}$ iff $\mathbf{\Delta}, \mathbf{Q} < \mathbf{R}$.

It is easily seen that $\approx$ is indeed an equivalence relation.

Ground-theoretic equivalence is closely related to identity on bilateral and unilateral contents. Evidently, the identity of bilateral contents implies their (ground-theoretic) equivalence. Indeed, because we have defined ground on bilateral contents without regard to negative content, two propositions will be equivalent already if they have the same *positive* content. This gives us a first substantive result about equivalence. For the DeMorgan laws are easily shown to hold for identity of positive content, and they therefore also hold for $\approx$:

(DeMorgan 1): $\neg(\mathbf{P} \wedge \mathbf{Q}) \approx \neg\mathbf{P} \vee \neg\mathbf{Q}$
(DeMorgan 2): $\neg(\mathbf{P} \vee \mathbf{Q}) \approx \neg\mathbf{P} \wedge \neg\mathbf{Q}$

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38 Further structural principles about ground might of course be considered, and implemented in the form of suitable constraints on propositions. The most significant ones among them may be the various versions of the claim that ground is well-founded. For an illuminating discussion of the various possible interpretations of the claim, see Dixon (2016). See Litland (2016a) for an argument for an instance of non-well-founded grounding.

39 I borrow the term from (Fine, 2012b: pp. 63, 67), who does not explicitly define it but seems to use it in at least roughly the same sense.

Assuming irreflexivity, the relation between identity of positive content and $\approx$ turns out to be even tighter: $\approx$ is *equivalent* to identity of positive content.[40]

($\approx$-Equivalence): $\mathbf{P} \approx \mathbf{Q}$ iff $\mathbf{P}^+ = \mathbf{Q}^+$

Using this fact, it is easy to show that $\approx$ is preserved under $\wedge$, $\vee$, and $\neg\neg$:

($\approx \wedge$): If $\mathbf{P} \approx \mathbf{Q}$ then $\mathbf{P} \wedge \mathbf{R} \approx \mathbf{Q} \wedge \mathbf{R}$ and $\mathbf{R} \wedge \mathbf{P} \approx \mathbf{R} \wedge \mathbf{Q}$
($\approx \vee$): If $\mathbf{P} \approx \mathbf{Q}$ then $\mathbf{P} \vee \mathbf{R} \approx \mathbf{Q} \vee \mathbf{R}$ and $\mathbf{R} \vee \mathbf{P} \approx \mathbf{R} \vee \mathbf{Q}$
($\approx \neg\neg$): If $\mathbf{P} \approx \mathbf{Q}$ then $\neg\neg\mathbf{P} \approx \neg\neg\mathbf{Q}$

However, it can be shown that $\approx$ is *not* preserved under $\neg$. The DeMorgan equivalents constitute a relatively simple counter-example. To get an idea why, note that although $\neg(\mathbf{P} \wedge \mathbf{Q}) \approx \neg\mathbf{P} \vee \neg\mathbf{Q}$, negating the right-hand side yields $\neg(\neg\mathbf{P} \vee \neg\mathbf{Q})$, which by the other DeMorgan law is equivalent to $\neg\neg\mathbf{P} \wedge \neg\neg\mathbf{Q}$. This, however, is not in general equivalent to $\neg\neg(\mathbf{P} \wedge \mathbf{Q})$, that is, the negation of the left-hand side of the original equivalence. For while the former is always grounded by $\neg\neg\mathbf{P}, \neg\neg\mathbf{Q}$, this is not in general true of $\neg\neg(\mathbf{P} \wedge \mathbf{Q})$.

We should also like to know under what conditions the various kinds of truth-functionally complex propositions are equivalent to one another. The answer to this depends on the general conditions under which V is taken to map two sequences to the same mode. So far, we have committed to the thesis that given any sequence of propositions, there is at most one mode of verifying via that sequence. So any mode which is a mode of verifying via some sequence of propositions is uniquely identified by that sequence. This puts an upper bound on the fineness of grain with which we may individuate modes: they are at most as finely individuated as the corresponding sequences of propositions.

We have also assumed that two sequences correspond to the same mode *only if* the sequences correspond to the same *set*, i.e. if the same propositions belong to both. This puts a lower bound on the fineness of grain with which we may individuate modes: they are at least as finely individuated as the *sets* determined by their corresponding sequences. There is at least one natural intermediate option, which is to abstract from the order of the propositions in a sequence corresponding to a mode, but not from repetitions. On this view, modes are exactly as finely individuated as the *multi-sets* determined by their corresponding sequences.

We shall say that the via-function V is *sequential* iff: $V(\gamma) = V(\delta)$ just in case $\gamma = \delta$; *semi-extensional* iff: $V(\gamma) = V(\delta)$ just in case γ and δ determine the same multi-set; and *extensional* iff: $V(\gamma) = V(\delta)$ just in case γ and δ determine the same set. It is

[40] For assume $\mathbf{P} \approx \mathbf{Q}$. Note that $\mathbf{P} < \neg\neg\mathbf{P}$, so $\mathbf{Q} < \neg\neg\mathbf{P}$, so $\mathbf{Q} \leq \mathbf{P}$, and hence either $\mathbf{Q} < \mathbf{P}$ or $\mathbf{Q}^+ = \mathbf{P}^+$. So if $\mathbf{Q}^+ \neq \mathbf{P}^+$, it follows that $\mathbf{Q} < \mathbf{P}$. By $\mathbf{P} \approx \mathbf{Q}$, we may infer $\mathbf{P} < \mathbf{P}$, contrary to the assumption of irreflexivity.

beyond the scope of this paper to determine, in terms of the relationships between the component propositions, the exact conditions under which truth-functionally complex propositions will be equivalent under each of these views. I shall here confine myself to some observations concerning the most distinctive features of the three approaches.

If V is sequential, then it will not in general be the case that $\mathbf{P} \wedge \mathbf{Q} \approx \mathbf{Q} \wedge \mathbf{P}$, or that $\mathbf{P} \vee \mathbf{Q} \approx \mathbf{Q} \vee \mathbf{P}$. For suppose $\mathbf{P}$ and $\mathbf{Q}$ are fundamental propositions, i.e. propositions whose positive components contain no derivative modes, and suppose $\mathbf{P}^+ \neq \mathbf{Q}^+$. For the case of conjunction, note that $\mathbf{P}^+ \wedge \mathbf{Q}^+$ and $\mathbf{Q}^+ \wedge \mathbf{P}^+$ will then each include exactly one derivative mode, namely $V\langle \mathbf{P}^+, \mathbf{Q}^+ \rangle$ and $V\langle \mathbf{Q}^+, \mathbf{P}^+ \rangle$, respectively. Since $\mathbf{P} \neq \mathbf{Q}$, $\langle \mathbf{P}, \mathbf{Q} \rangle \neq \langle \mathbf{Q}, \mathbf{P} \rangle$. By sequentiality of V, $V\langle \mathbf{P}^+, \mathbf{Q}^+ \rangle \neq V\langle \mathbf{Q}^+, \mathbf{P}^+ \rangle$, and hence $\mathbf{P}^+ \wedge \mathbf{Q}^+ \neq \mathbf{Q}^+ \wedge \mathbf{P}^+$, so by ($\approx$-Equivalence), $\mathbf{P} \wedge \mathbf{Q} \not\approx \mathbf{Q} \wedge \mathbf{P}$. Similar considerations establish the point for disjunction. However, if V is semi-extensional or extensional, these equivalences will hold. We consider only conjunction. Every mode in $\mathbf{P}^+ \wedge \mathbf{Q}^+$ may be written $V(\gamma \frown \delta)$ where $V(\gamma) \in \mathbf{P}^+$ and $V(\delta) \in \mathbf{Q}^+$. But then $\mathbf{Q}^+ \wedge \mathbf{P}^+$ includes $V(\delta \frown \gamma)$, and since $\gamma \frown \delta$ and $\delta \frown \gamma$ determine the same multi-set, and therefore the same set, the modes are identical.

A distinctive feature of the extensional approach is that it yields some general equivalences between the values of *different* truth-functional operations. In particular, for any closed $\mathbf{P}$, we have $\mathbf{P} \wedge \mathbf{P} \approx \mathbf{P} \vee \mathbf{P} \approx \neg\neg\mathbf{P}$. Indeed, the contents in question will be fully identical.[41] Now call a bilateral proposition conjunctive, disjunctive, or negative, according as it is the value of the operation of conjunction, disjunction, or negation on bilateral contents. Our observation then shows that on the extensional approach, there is no clear divide among the propositions that are conjunctive, disjunctive, or negative, but that these categories overlap. This is significant because it calls into question at least one interpretation of Correia's claim that the acceptance of the unrestricted introduction principles for ground commits us to a conceptual, or representational conception of the relata of ground. For one good sense we could give to the notion of a representational conception of content is that it applies just in case contents are individuated in terms of the concepts invoked in expressing them. But this would appear to imply, at the very least, that there is a mutually exclusive division into conjunctive and disjunctive contents.

Another striking feature of the extensional approach is that, assuming a modest form of the transitivity of ground, it allows us to characterize binary ground, weak or strict, purely in terms of disjunction, equivalence, and inequivalence. For $\mathbf{P} < \mathbf{Q}$ is equivalent

41 The crucial observation is that if modes are extensional, the operation of fusion on the modes is idempotent, i.e. $m \sqcup m = m$. This renders the operations on closed contents of $\sqcup$ and $+$ idempotent, from which the above identities follow straightforwardly.

to $\mathbf{P} \leq \mathbf{Q}$ and $\mathbf{P} \not\approx \mathbf{Q}$, and the condition that $\mathbf{P} \leq \mathbf{Q}$ now turns out equivalent to the condition that $\mathbf{P} \vee \mathbf{Q} \approx \mathbf{Q} \vee \mathbf{Q}$. To see this, note that the extensional approach renders equivalent the propositions $\mathbf{P}, \mathbf{Q}$ iff they have the same strict full grounds, assuming they have some strict full grounds at all. Then given transitivity, any strict full ground of $\mathbf{P}$ is a strict full ground of $\mathbf{Q}$ if $\mathbf{P} \leq \mathbf{Q}$, and hence the strict full grounds of $\mathbf{P} \vee \mathbf{Q}$ are exactly the same as the strict full grounds of $\mathbf{Q} \vee \mathbf{Q}$. This is significant in part because the very same equivalence holds on the Finean truthmaker account of ground above, and in the logic of Correia (2010). Only on the latter views, the equivalence can be further simplified, owing to the identification of a proposition with its self-disjunction. So we see that on the extensional approach to modes, the view of ground that emerges stays remarkably close to the so-called worldly accounts.

6. Conclusion

In this paper, I have proposed a novel answer to the question what features of a sentence's content the notion of ground tracks: it tracks in what *modes* a content is verified. Roughly speaking, some truths Γ (strictly fully) ground a truth Q just in case verifying Γ is a mode of verifying Q. Based on this idea, I have presented an elementary formal theory of ground-theoretic content, on which a content encodes the information in what modes it may be verified or falsified. This theory, I maintain, is very well suited to serve as a framework within which a number of alternative views of ground may be articulated and examined. By way of defending this claim, I have presented a very natural and attractive account of the truth-functional operations and their interaction with ground, and I have shown how various possible views of the structural features of ground can be accommodated within my framework. Finally, I have described three views of the identity conditions of modes and some of their implications for the logic of ground, highlighting some surprising and intriguing features of the most coarse-grained of the views.

A natural next step is now to try and determine the exact pure and propositional logics of ground corresponding to a range of competing views of ground that may be implemented in the framework. In addition, various kinds of extensions of the framework may be attempted, for example by treating quantification, modal operators, and perhaps iterated ground, i.e. the grounds of truths of the form $\Gamma < P$. Already in its present form, however, the framework represents a significant step forward on the way towards a satisfactory and comprehensive theory of ground, and of ground-theoretic content.

Appendix

We begin with the definition of a mode-space.

Definition (Mode-Spaces) A *mode-space* is a pair $\langle M, V\rangle$ such that

(1) M is non-empty
(2) V is a non-empty, partial function taking sequences of non-empty subsets of M into members of M
(3) the domain of V is closed under non-empty subsequences and countable concatenations of sequences
(4) $V(\gamma_1{}^\frown\gamma_2{}^\frown\ldots) = V(\delta_1{}^\frown\delta_2{}^\frown\ldots)$ whenever $V(\gamma_1) = V(\delta_1), V(\gamma_2) = V(\delta_2), \ldots$

Given a fixed mode-space $\langle M, V\rangle$, we call a *mode* any member of M. Any mode which is the value of V for some argument is called *derivative*, every other mode is called *fundamental*. We write M^D (M^F) for the set of derivative (fundamental) modes. Any subset of M will be called a *content*, and their set will be denoted by $\mathscr{C}$. The contents containing only derivative modes will themselves be called derivative, and the other contents will be called fundamental. We write $\mathscr{C}^D$ ($\mathscr{C}^F$) for the set of derivative (fundamental) contents. Any sequence for which V is defined is called a *via-sequence*. Since the domain of V is closed under non-empty subsequences, for any content P that is an element of some via-sequence, there is also the via-sequence $\langle P\rangle$ corresponding to the mode of verifying via P. We shall call any such content *raisable* and denote their set by $\mathscr{R}$. We call *a ground-set* of a derivative mode m any set of contents that underlies a content-sequence γ with $V(\gamma) = m$.

We say that a derivative mode m is *a fusion of* the sequence of derivative modes $\langle m_1, m_2, \ldots\rangle$ iff there are via-sequences $\gamma_1, \gamma_2, \ldots$ such that $V(\gamma_1) = m_1$, $V(\gamma_2) = m_2$, ..., and $m = V(\gamma_1{}^\frown\gamma_2{}^\frown\ldots)$.[42] By the third constraint on mode-spaces, fusions will always exist, and by the fourth constraint, they will be unique. For the sequence of derivative modes $\langle m_1, m_2, \ldots\rangle$, we write its fusion as $\bigsqcup\langle m_1, m_2, \ldots\rangle$ and also as $m_1 \sqcup m_2 \sqcup \ldots$. We note a first central lemma (the proof is elementary):

Lemma 1. *If* $\Gamma_1, \Gamma_2, \ldots$ *are ground-sets of* $m_1, m_2, \ldots$*, respectively, then* $\Gamma_1 \cup \Gamma_2 \cup \ldots$ *is a ground-set of* $m_1 \sqcup m_2 \sqcup \ldots$

For convenience, we repeat the definitions of the grounding relationships and of the truthfunctions.

Definition Let $\Gamma \subseteq \mathscr{C}$ and $P \in \mathscr{C}$. Then

$\Gamma < P \quad :\leftrightarrow$ for some mode $m \in P$, Γ is a ground-set of m
$Q \prec P \quad :\leftrightarrow$ for some $m \in P$, Q is a member of a ground-set of m
$\Gamma \leq P \quad :\leftrightarrow \Gamma = \{P\}$ or $\Gamma < P$ or $\Gamma \backslash \{P\} < P$

[42] Note that somewhat unusally, our operation of fusion applies to sequences of modes and is thereby potentially sensitive to order and repetition of the modes being fused.

$Q \preceq P \quad :\leftrightarrow Q = P$ or $Q \prec P$

As before, we write $\Gamma < \{P_1, P_2, \ldots\}$ to abbreviate that for some $\Gamma_1, \Gamma_2, \ldots, \Gamma = \Gamma_1 \cup \Gamma_2 \cup \ldots$ and $\Gamma_1 < P_1$, $\Gamma_2 < P_2$, $\ldots$, and similarly for the case of $\Gamma \leq \{P_1, P_2, \ldots\}$.

Definition For $P, Q \in \mathscr{C}^D$:

$$P \sqcup Q \quad := \quad \{m \sqcup n : m \in P \text{ and } n \in Q\}$$
$$P + Q \quad := \quad (P \cup Q) \cup (P \sqcup Q)$$

Definition For $P, Q \in \mathscr{R}$:

$$\uparrow P \quad := \quad \{V\langle P\rangle\} \text{ if } P \cap M^D \text{ is empty, } \{V\langle P\rangle\} + (P \cap M^D) \text{ otherwise}$$
$$P \wedge Q \quad := \uparrow P \sqcup \uparrow Q$$
$$P \vee Q \quad := \uparrow P + \uparrow Q$$

We shall mainly be interested in mode-spaces in which the set of raisable contents is closed under these operations.

Definition A mode-space is called *complete* iff $P \wedge Q \in \mathscr{R}$, $P \vee Q \in \mathscr{R}$, and $\uparrow P \in \mathscr{R}$ whenever $P, Q \in \mathscr{R}$.

Lemma 2. *(Introduction Lemma)*
In any complete mode-space, for $\Gamma \subseteq \mathscr{R}$ and $P, Q \in \mathscr{R}$:

(1) *If $\Gamma \leq P$, then $\Gamma < \uparrow P$.*
(2) *If $\Gamma < \{P, Q\}$, then $\Gamma < P \sqcup Q$.*
(3) *If $\Gamma < P$ or $\Gamma < Q$ or $\Gamma < \{P, Q\}$, then $\Gamma < P + Q$*
(4) *If $\Gamma \leq \{P, Q\}$, then $\Gamma < P \wedge Q$*
(5) *If $\Gamma \leq P$ or $\Gamma \leq Q$ or $\Gamma < \{P, Q\}$, then $\Gamma < P \vee Q$.*

Proof. For 1: Suppose $\Gamma \leq P$. Then either (i) $\Gamma = \{P\}$, or (ii) $\Gamma < P$, or (iii) $\Gamma \backslash \{P\} < P$. Suppose (i). By definition of $\uparrow$, $V\langle P\rangle \in \uparrow P$. Since $\{P\}$ is the set underlying $\langle P\rangle$, it is a ground-set of $V\langle P\rangle$, so $\Gamma < P$. Suppose (ii). Then Γ is a ground-set of some mode $m \in P$. But then by definition of $\uparrow$it follows that $m \in \uparrow P$, and hence $\Gamma < P$. Suppose (iii). Then for some $m \in P$, $\Gamma \backslash \{P\}$ is a ground-set of m. But then by definition of $\uparrow$it follows that $m \sqcup V\langle P\rangle \in \uparrow P$. By lemma 1, $\Gamma \backslash \{P\} \cup \{P\} = \Gamma$ is a ground-set of $m \sqcup V\langle P\rangle$, hence $\Gamma < P$.

For 2: Suppose $\Gamma < \{P, Q\}$. Let $\Gamma_P < P$ and $\Gamma_Q < Q$ with $\Gamma_P \cup \Gamma_Q = \Gamma$. Then Γ_P is a ground-set of some mode $m_P \in P$ and Γ_Q is a ground-set of some mode $m_Q \in Q$. By definition of $\sqcup$, $m_P \sqcup m_Q \in P \sqcup Q$, and by lemma 1, $\Gamma_P \cup \Gamma_Q = \Gamma$ is a ground-set of $m_P \sqcup m_Q$, hence $\Gamma < P \sqcup Q$.

For 3: Suppose $\Gamma < P$. Then there is a mode $m \in P$ of which Γ is a ground-set. By definition of $+$, the same mode is included in $P + Q$, hence $\Gamma < P + Q$. Suppose $\Gamma < Q$.

Then by the same reasoning, $\Gamma < P + Q$. Finally, suppose $\Gamma < \{P, Q\}$. Then by part 2., $\Gamma < P \sqcup Q$, and hence by definition of $+$, $\Gamma < P + Q$.

For 4: Suppose $\Gamma \leq \{P, Q\}$. Let $\Gamma_P \leq P$ and $\Gamma_Q \leq Q$ with $\Gamma_P \cup \Gamma_Q = \Gamma$. By part 1., $\Gamma_P <\uparrow P$ and $\Gamma_Q <\uparrow Q$, hence $\Gamma < \{\uparrow P, \uparrow Q\}$, and by part 2., $\Gamma < P \wedge Q$.

For 5: If $\Gamma \leq P$ or $\Gamma \leq Q$, then by part 2., $\Gamma <\uparrow P$ or $\Gamma <\uparrow Q$. If $\Gamma \leq \{P, Q\}$, then by the reasoning in part 4., $\Gamma < \{\uparrow P, \uparrow Q\}$. So by part 3., $\Gamma < P \vee Q$. □

Definition A mode-space is called *constrained* iff $V(\gamma) = V(\delta)$ only if the same ground-set corresponds to γ and δ.

In a constrained mode-space, every derivative mode m corresponds to a unique ground-set, which we denote by $|m|$.

Lemma 3. *(Elimination Lemma)*
In any constrained and complete mode-space, for $\Gamma \subseteq \mathscr{R}$ and $P, Q \in \mathscr{R}$:

(1) If $\Gamma <\uparrow P$, then $\Gamma \leq P$
(2) If $\Gamma < P \sqcup Q$, then $\Gamma < \{P, Q\}$
(3) If $\Gamma < P + Q$, then $\Gamma < P$ or $\Gamma < Q$ or $\Gamma < \{P, Q\}$
(4) If $\Gamma < P \wedge Q$, then $\Gamma \leq \{P, Q\}$
(5) If $\Gamma < P \vee Q$, then $\Gamma \leq P$ or $\Gamma \leq Q$ or $\Gamma \leq \{P, Q\}$

Proof. For 1: Suppose $\Gamma <\uparrow P$. Let $m \in \uparrow P$ with $\Gamma = |m|$. By definition of $\uparrow$, there are three cases. (i) $m \in P \cap M^D$. Then $\Gamma < P$ and hence $\Gamma \leq P$. (ii) $m = V\langle P\rangle$. Then $\Gamma = |V\langle P\rangle| = \{P\}$, so again $\Gamma \leq P$. (iii) $m = V\langle P\rangle \sqcup n$ for some $n \in P \cap M^D$. Since $n \in P$, $|n| < P$. By lemma 1, $\Gamma = |n| \cup \{P\}$. Then either $|n| = \Gamma$ or $|n| = \Gamma \backslash \{P\}$, so either $\Gamma < P$ or $\Gamma \backslash \{P\} < P$, and hence $\Gamma \leq P$.

For 2: Suppose $\Gamma < P \sqcup Q$. Let $|m| = \Gamma$ and $m \in P \sqcup Q$. Then $m = m_P \sqcup m_Q$ for some $m_P \in P$ and $m_Q \in Q$. So $|m_P| < P$ and $|m_Q| < Q$. But $\Gamma = |m_P| \cup |m_Q$, and hence $\Gamma < \{P, Q\}$.

For 3: Suppose $\Gamma < P + Q$. Then it is immediate from the definition of $+$ that $\Gamma < P$ or $\Gamma < Q$ or $\Gamma < P \sqcup Q$, which by part 2. implies $\Gamma < \{P, Q\}$.

For 4: Suppose $\Gamma < P \wedge Q$. By part 2., $\Gamma < \{\uparrow P, \uparrow Q\}$. So let $\Gamma_P <\uparrow P$ and $\Gamma_Q <\uparrow Q$ with $\Gamma = \Gamma_P \cup \Gamma_Q$. By part 1., $\Gamma_P \leq P$ and $\Gamma_Q \leq Q$, hence $\Gamma \leq \{P, Q\}$.

For 5: Suppose $\Gamma < P \vee Q$. By part 3., there are three cases. (i) $\Gamma <\uparrow P$. Then by part 1., $\Gamma \leq P$. (ii) $\Gamma <\uparrow Q$. Then by part 1. again, $\Gamma \leq Q$. (iii) $\Gamma \leq \{\uparrow P, \uparrow Q\}$. Then by the reasoning in part 4., $\Gamma \leq \{P, Q\}$. □

We move on to the case of bilateral contents. We define the truth-functional operations on bilateral contents as well as the notion of ground-theoretic equivalence ($\approx$).

Definition For $\mathbf{P}, \mathbf{Q} \in \mathscr{R} \times \mathscr{R}$:

$$\begin{array}{lll} \neg\mathbf{P} & := & \langle \mathbf{P}^-, \uparrow\mathbf{P}^+ \rangle \\ \mathbf{P}\wedge\mathbf{Q} & := & \langle \mathbf{P}^+ \wedge \mathbf{Q}^+, \mathbf{P}^- \vee \mathbf{Q}^- \rangle \\ \mathbf{P}\vee\mathbf{Q} & := & \langle \mathbf{P}^+ \vee \mathbf{Q}^+, \mathbf{P}^- \wedge \mathbf{Q}^- \rangle \end{array}$$

Definition For $\mathbf{P},\mathbf{Q} \in \mathscr{R}\times\mathscr{R}$:
$\mathbf{P} \approx \mathbf{Q} :\leftrightarrow$ for all $\mathbf{\Gamma}$: $\mathbf{\Gamma} < \mathbf{P}$ iff $\mathbf{\Gamma} < \mathbf{Q}$, and for all $\mathbf{\Delta}$ and $\mathbf{R}$: $\mathbf{\Delta}, \mathbf{P} < \mathbf{R}$ iff $\mathbf{\Delta}, \mathbf{Q} < \mathbf{R}$.

Recall that ground between bilateral contents is defined simply as ground between the positive components. As a result, bilateral contents will be *ground-theoretically equivalent* (written $\approx$) provided their positive components are the same.

Lemma 4. *(DeMorgan) For* $\mathbf{P},\mathbf{Q} \in \mathscr{R}\times\mathscr{R}$*:*

(1) $\neg(\mathbf{P}\wedge\mathbf{Q}) \approx \neg\mathbf{P}\vee\neg\mathbf{Q}$
(2) $\neg(\mathbf{P}\vee\mathbf{Q}) \approx \neg\mathbf{P}\wedge\neg\mathbf{Q}$

Proof. By application of the definitions.
For 1: $(\neg(\mathbf{P}\wedge\mathbf{Q}))^+ = (\mathbf{P}\wedge\mathbf{Q})^- = \mathbf{P}^- \vee \mathbf{Q}^- = (\neg\mathbf{P})^+ \vee (\neg\mathbf{Q})^+ = (\neg\mathbf{P}\vee\neg\mathbf{Q})^+$
For 2: $(\neg(\mathbf{P}\vee\mathbf{Q}))^+ = (\mathbf{P}\vee\mathbf{Q})^- = \mathbf{P}^- \wedge \mathbf{Q}^- = (\neg\mathbf{P})^+ \wedge (\neg\mathbf{Q})^+ = (\neg\mathbf{P}\wedge\neg\mathbf{Q})^+$ □

As an immediate consequence of the definition of ground on bilateral contents and the previous lemmata, we obtain

Theorem 5. *(Truthfunctions and Ground, Introduction)*
In any complete mode-space, for $\mathbf{\Gamma} \subseteq \mathscr{R}\times\mathscr{R}$ *and* $\mathbf{P}, \mathbf{Q} \in \mathscr{R}\times\mathscr{R}$*:*

(1) If $\mathbf{\Gamma} \leq \{\mathbf{P}, \mathbf{Q}\}$*, then* $\mathbf{\Gamma} < \mathbf{P}\wedge\mathbf{Q}$
(2) If $\mathbf{\Gamma} \leq \mathbf{P}$ *or* $\mathbf{\Gamma} \leq \mathbf{Q}$ *or* $\mathbf{\Gamma} < \{\mathbf{P}, \mathbf{Q}\}$*, then* $\mathbf{\Gamma} < \mathbf{P}\vee\mathbf{Q}$.
(3) If $\mathbf{\Gamma} \leq \mathbf{P}$*, then* $\mathbf{\Gamma} < \neg\neg\mathbf{P}$
(4) If $\mathbf{\Gamma} \leq \neg\mathbf{P}$ *or* $\mathbf{\Gamma} \leq \neg\mathbf{Q}$ *or* $\mathbf{\Gamma} \leq \{\neg\mathbf{P}, \neg\mathbf{Q}\}$*, then* $\mathbf{\Gamma} < \neg(\mathbf{P}\wedge\mathbf{Q})$
(5) If $\mathbf{\Gamma} \leq \{\neg\mathbf{P}, \neg\mathbf{Q}\}$*, then* $\mathbf{\Gamma} < \neg(\mathbf{P}\vee\mathbf{Q})$

Theorem 6. *(Truthfunctions and Ground, Elimination)*
In any complete and constrained mode-space, for $\mathbf{\Gamma} \subseteq \mathscr{R}\times\mathscr{R}$ *and* $\mathbf{P}, \mathbf{Q} \in \mathscr{R}\times\mathscr{R}$*:*

(1) If $\mathbf{\Gamma} < \mathbf{P}\wedge\mathbf{Q}$*, then* $\mathbf{\Gamma} \leq \{\mathbf{P}, \mathbf{Q}\}$
(2) If $\mathbf{\Gamma} < \mathbf{P}\vee\mathbf{Q}$*, then* $\mathbf{\Gamma} \leq \mathbf{P}$ *or* $\mathbf{\Gamma} \leq \mathbf{Q}$ *or* $\mathbf{\Gamma} \leq \{\mathbf{P}, \mathbf{Q}\}$
(3) If $\mathbf{\Gamma} < \neg\neg\mathbf{P}$*, then* $\mathbf{\Gamma} \leq \mathbf{P}$
(4) If $\mathbf{\Gamma} < \neg(\mathbf{P}\wedge\mathbf{Q})$*, then* $\mathbf{\Gamma} \leq \neg\mathbf{P}$ *or* $\mathbf{\Gamma} \leq \neg\mathbf{Q}$ *or* $\Gamma \leq \{\neg\mathbf{P}, \neg\mathbf{Q}\}$
(5) If $\mathbf{\Gamma} < \neg(\mathbf{P}\vee\mathbf{Q})$*, then* $\mathbf{\Gamma} \leq \{\neg\mathbf{P}, \neg\mathbf{Q}\}$

We turn now to the structural properties of ground.

Definition A proposition P is

- *irreflexive* iff: P does not occur in γ whenever $V(\gamma) \in P$,

- *closed* iff: P contains a mode with ground-set $\Gamma_1 \cup \Gamma_2 \cup \ldots$ whenever P contains modes with ground-sets $\Gamma_1, \Gamma_2, \ldots$,
- *transitive* iff: P includes a mode with ground-set Γ, Δ whenever P includes a mode with ground-set Δ, Q and Q includes a mode with ground-set Γ.
- *normal* iff: irreflexive, closed, and transitive.

Theorem 7. *(Structural Principles for Normal Propositions) In any constrained mode-space, for normal propositions $P, P_1, P_2, \ldots, Q, R$ and sets of normal propositions $\Gamma, \Gamma_1, \Gamma_2, \ldots, \Delta$:*

(1) $P \not< P$
(2) *If* $\Gamma_1 < P$, $\Gamma_2 < P, \ldots,$ *then* $\Gamma_1, \Gamma_2, \ldots < P$.
(3) *If* $\Gamma \leq P$ *and* $Q \prec P$ *for all* $Q \in \Gamma$, *then* $\Gamma < P$.
(4) *If* $\Gamma, P \leq P$, *then* $\Gamma \leq P$.
(5) *If* $\Gamma_1 \leq P$, $\Gamma_2 \leq P, \ldots,$ *then* $\Gamma_1 \cup \Gamma_2 \cup \ldots \leq P$.
(6) *If* $\Gamma < P$ *and* $\Delta, P < Q$, *then* $\Gamma, \Delta < Q$.
(7) *If* $\Gamma_1 \leq P_1$, $\Gamma_2 \leq P_2, \ldots,$ *and* $P_1, P_2, \ldots \leq Q$ *then* $\Gamma_1, \Gamma_2, \ldots \leq Q$
(8) *If* $P \preceq Q$ *and* $Q \prec R$ *then* $P \prec R$
(9) *If* $P \prec Q$ *and* $Q \preceq R$ *then* $P \prec R$
(10) *If* $P \preceq Q$ *and* $Q \preceq R$ *then* $P \preceq R$

Proof. 1.–4., and 6. were established in section 5.4 above.

For 5., suppose $\Gamma_1 \leq P$, $\Gamma_2 \leq P, \ldots$ Consider all the Γ_i which are distinct from $\{P\}$. By definition of $\leq$ and 1., $\Gamma_i \backslash \{P\} < P$ for any such Γ_i. If there are any such Γ_i, let Γ' be the result of removing P from their union. By 2., $\Gamma' < P$, so $\Gamma_1 \cup \Gamma_2 \cup \ldots = \Gamma' \cup \{P\} \leq P$. If there are no such Γ_i, then $\Gamma_1 \cup \Gamma_2 \cup \ldots = \{P\} \leq P$.

For 7., suppose $\Gamma_1 \leq P_1$, $\Gamma_2 \leq P_2, \ldots,$ and $P_1, P_2, \ldots \leq Q$. We confine ourselves to showing that $\Gamma_1 \cup \{P_2, \ldots\} \leq Q$ follows. By applying the same reasoning repeatedly and making use of the reflexivity of $\leq$, we may establish the desired conclusion. Now either (a) $\Gamma_1 = \{P_1\}$, or (b) $\Gamma_1 < P_1$, or (c) $\{P_1\} \subset \Gamma_1$ and $\Gamma_1 \backslash \{P_1\} < P_1$. If (a), then our intended result $\Gamma_1 \cup \{P_2, \ldots\} \leq Q$ follows immediately. Suppose that (b). Then if (b1) $P_1 = Q$, we have $\Gamma_1 < Q$ and hence $\Gamma_1 \leq Q$. Moreover, by 4., we have $\{P_2, \ldots\} \leq Q$. So by 5., $\Gamma_1 \cup \{P_2, \ldots\} \leq Q$ follows. But if (b2) $P_1 \neq Q$, then $P_1 \in \{P_1, P_2, \ldots\} \backslash \{Q\}$ and $\{P_1, P_2, \ldots\} \backslash \{Q\} < Q$, so by 6., $\Gamma_1 \cup \{P_2, \ldots\} \backslash \{Q\} < Q$, hence $\Gamma_1 \cup \{P_2, \ldots\} \leq Q$. Suppose finally that (c). Then if (c1) $P_1 = Q$, $\Gamma_1 \leq Q$, and so $\Gamma_1 \cup \{P_2, \ldots\} \leq Q$ follows as in case (b1). But if (c2) $P_1 \neq Q$, then by similar reasoning as in case (b2), $\Gamma_1 \backslash \{P_1\} \cup \{P_1, P_2, \ldots\} = \Gamma_1 \cup \{P_2, \ldots\} \leq Q$.

For 8., suppose $P \preceq Q$ and $Q \prec R$. If $P = Q$, then $P \prec R$ follows immediately. So suppose $P \neq Q$, and hence $P \prec Q$. Let $m \in Q$ be such that $P \in |m|$, and let $n \in R$ be such that $Q \in |n|$. Then $|m| \cup \{P\} < Q$ and $|n| \cup \{Q\} < R$, so by 6., $|n| \cup |m| \cup \{P\} < R$, so $P \prec R$.

For 9., suppose $P \prec Q$ and $Q \preceq R$. If $Q = R$, then $P \prec R$ follows immediately. So suppose $Q \neq R$, and hence $Q \prec R$. Then by the same reasoning as before, $P \prec R$.

For 10., suppose $P \preceq Q$ and $Q \preceq R$. If either $P \prec Q$ or $Q \prec R$, then it follows by the previous results that $P \prec R$, and hence $P \preceq R$. If neither $P \prec Q$ nor $Q \prec R$, then $P = Q$ and $Q = R$, hence $P = R$, and thus again $P \preceq R$. □

Together with the Subsumption principles and the Identity principle $P \leq P$ for weak ground established in 5.2, parts 1., 3., 7.-10. correspond to the basic rules of the logic proposed in (Fine, 2012c), which is thereby shown to be sound with respect to the class of normal propositions in a constrained mode-space.

We call a bilateral content irreflexive, closed, transitive, or normal, just in case its positive component has the relevant property. We wish then to show that any truth-functional combinations of normal propositions are themselves normal. By the definitions of the truth-functional operations, it suffices to show that for unilateral contents, normality is preserved under $\wedge, \vee$, and $\uparrow$.

Theorem 8. *For $P, Q \in \mathscr{R}$, in some constrained mode-space:*
If P and Q are normal, then so are $P \wedge Q$, $P \vee Q$, and $\uparrow P$.

Proof. By reducing failures for $P \wedge Q$, $P \vee Q$, $\uparrow P$ of the characteristic principles of irreflexivity, amalgamation, and transitivity (1., 2., and 6. in theorem 7) to failures of the corresponding principles for P or Q. We give the proof for $P \wedge Q$; the other cases may be established by parallel means.

For irreflexivity, suppose that $P \wedge Q \prec P \wedge Q$. Then for some Γ: $\Gamma, P \wedge Q < P \wedge Q$. Then by the Elimination Lemma, $\Gamma, P \wedge Q \leq \{P, Q\}$, and hence either (i) $P \wedge Q \preceq P$ or (ii) $P \wedge Q \preceq Q$. Suppose (i). Then since $P \prec P \wedge Q$, it follows by transitivity of P that $P \prec P$, in contradiction to P's irreflexivity.

For amalgamation, suppose that $\Gamma_1 < P \wedge Q$, $\Gamma_2 < P \wedge Q$, ... For each Γ_i: $\Gamma_i \leq \{P, Q\}$. So let $\Gamma_{i_P} \leq P$ and $\Gamma_{i_Q} \leq Q$ with $\Gamma_i = \Gamma_{i_P} \cup \Gamma_{i_Q}$ for all i. Then by weak ground amalgamation for P and Q, $\Gamma_{1_P}, \Gamma_{2_P}, \ldots \leq P$, and $\Gamma_{1_Q}, \Gamma_{2_Q}, \ldots \leq Q$, so $\Gamma_1, \Gamma_2, \ldots \leq \{P, Q\}$, and hence $\Gamma_1, \Gamma_2, \ldots < P \wedge Q$.

For transitivity, suppose that $\Delta, R < P \wedge Q$ and $\Gamma < R$. Then $\Delta, R \leq \{P, Q\}$. So we may write Δ, R as the union of weak full grounds of respectively P and Q. R will be a member of at least one of these. By replacing R with Γ, using the transitivity of P and Q, we may infer $\Gamma, \Delta < \{P, Q\}$, and hence $\Gamma, \Delta < P \wedge Q$. □

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