The Logic of Ground

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  - In symbols: $P \prec P \wedge Q$ and $Q \prec P \wedge Q$.
- Ground is irreflexive: $P \wedge Q \nprec P \wedge Q$. (However, a weakened, reflexive form of ground is often also recognized.)
- Partial ground requires relevance: Usually $P, Q, R \nprec P \wedge Q$.

Pure Logic

- Subsumption Principles
  - $\Gamma < C \Rightarrow \Gamma \leq C$
  - $\Gamma, A < C \Rightarrow A \prec C$
  - $\Gamma, A \leq C \Rightarrow A \preceq C$
  - $A \prec C \Rightarrow A \preceq C$
- Ir/reflexivity Principles
  - $\top \Rightarrow C \leq C$
  - $C \prec C \Rightarrow \bot$
- Transitivity Principles
  - $\Gamma \leq A;\ \Delta, A \leq C \Rightarrow \Delta, \Gamma \leq C$
  - $A \preceq B;\ B \preceq C \Rightarrow A \preceq C$
  - $A \prec B;\ B \preceq C \Rightarrow A \prec C$
  - $A \preceq B;\ B \prec C \Rightarrow A \prec C$
- Reverse Subsumption
  - $\Gamma \leq C;\ A \prec C$ for all $A \in \Gamma \Rightarrow \Gamma < C$

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- Then A's grounding C does not imply A, B's grounding C.

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- For $X \subseteq V$ and $P, Q \in V$:
  - $X \leq P$ iff $\bigsqcup X \subseteq P$
  - $Q \preceq P$ iff $X \cup \{Q\} \leq P$ for some $X \subseteq V$.
  - $Q \prec P$ iff $Q \preceq P$ and $Q \neq P$
  - $X < P$ iff $X \leq P$ and $P \not\preceq R$ for all $R \in X$

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- Informally,
  - the members of S are situations,
  - the set assigned to A is the set of situations verifying A
  - this set is not generally closed under $\sqsupseteq$ (relevance)

Propositional Logic

Let $\Gamma \leq \{A, B\}$ be true iff for some Γ_A, Γ_B: $\Gamma = \Gamma_A \cup \Gamma_B$ and $\Gamma_A \leq A$ and $\Gamma_B \leq B$.

- $\Gamma \leq A \Leftrightarrow \Gamma < \neg\neg A$
- $\Gamma \leq \{A, B\} \Leftrightarrow \Gamma < A \wedge B$
- $\Gamma \leq \{\neg A, \neg B\} \Leftrightarrow \Gamma < \neg(A \vee B)$
- $\Gamma \leq A$ or $\Gamma \leq B$ or $\Gamma \leq \{A, B\} \Leftrightarrow \Gamma < A \vee B$
- $\Gamma \leq \neg A$ or $\Gamma \leq \neg B$ or $\Gamma \leq \{\neg A, \neg B\} \Leftrightarrow \Gamma < \neg(A \wedge B)$

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Problem

- $A < \neg\neg A$ and $A < A \vee A$ and $A < A \wedge A$ but $A \not< A$
- A cannot plausibly be distinguished from $\neg\neg A$, $A \vee A$, and $A \wedge A$ in terms of their truthmakers

Semantics, Informal Idea

- Correlate sentences with the different *ways* or *modes* in which they may be verified (or falsified)
  - $A \vee B$ may be verified by verifying A
  - $A \vee B$ may be verified by verifying B
  - $A \wedge B$ may be verified by verifying $\{A, B\}$
  - ...

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  - $A \vee B$ may be verified by verifying A
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  - $A \wedge B$ may be verified by verifying $\{A, B\}$
  - ...
- Say that Γ strictly ground C iff verifying Γ is a way to verify C
  - $\neg\neg A$ may be verified by verifying A
  - $A \wedge A$ may be verified by verifying A
  - $A \vee A$ may be verified by verifying A
  - A may **not** be verified by verifying A

Semantics

- We interpret our language with respect to a pair (M, V), where
  - M is the (non-empty) set of modes
  - non-empty subsets of M are propositions
  - V maps some sets of propositions to modes
  - Informally, $V(\Gamma) = m$ means that m is the mode of verifying *by* verifying Γ
- Ground is interpreted as follows
  - $\Gamma < P$ iff $V(\Gamma) \in P$
  - $Q \prec P$ iff for some $\Delta \subseteq \wp(M)$: $\Delta \cup \{Q\} < P$
  - $\Gamma \leq P$ iff $\Gamma < P$ or $\Gamma = \{P\}$ or $\Gamma \backslash \{P\} < P$
  - $Q \preceq P$ iff $Q \prec P$ or $Q = P$
- Soundness of the Pure Logic above corresponds to natural structural constraints on V / the set of admissible propositions
- Completeness requires some additional rules

Semantics

- To interpret the propositional language
  - we define a natural operation of fusion $\sqcup$ on the modes
  - we extend it pointwise to propositions:
    $P \sqcup Q := \{m \sqcup n : m \in P, n \in Q\}$
  - we also use an operation of addition:
    $P + Q := P \cup Q \cup (P \sqcup Q)$
  - and of raising: $\uparrow P := (\{V(\{P\})\} + P) \cap ran(V)$
- we can then define conjunction and disjunction
  - $P \wedge Q := \uparrow P \sqcup \uparrow Q$
  - $P \vee Q := \uparrow P + \uparrow Q$
- For negation, we take a proposition to be a pair of sets of verifying and falsifying modes.
  - Let $\neg\langle P^+, P^-\rangle := \langle P^-, \uparrow P^+\rangle$
  - Let $(P \wedge Q)^- := (P \vee Q)^+$ and $(P \vee Q)^- := (P \wedge Q)^+$
- This yields soundness for the propositional rules.

Thanks